Transmitting and Sharing: a Truthful Double Auction for Cognitive Radio Networks

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Abstract—**The scarcity of spectrum channels resides in the limited bandwidth resource and the exploding demand from spectrum-based services and devices. To help ease this scarcity, the concept of cognitive radio networks (CRNs) is proposed, where licensed spectrum holders (primary users) may lease their channels to unlicensed users (secondary users). Many CRN auctions are thus designed to incentivize primary users (PUs) to share their idle channels with secondary users (SUs). Most of these auctions assume that a transmitting PU does not lease its channel to SUs; if it leases its channel to SUs, it does not transmit itself. To further utilize the resource, researchers have studied the scenario where a transmitting PU is allowed to lease its channels to SUs if the transmissions of the SUs do not undermine the transmission of the PU. However, the study assumes that there is only one PU who owns the licensed channels, whereas in practice, channels may be contributed by multiple PUs. This prevents the result of the study from being directly applied to the multi-PU scenario, as the potential competitions among the PUs are neglected. We extend the scenario to the CRN with multiple PUs and propose TDSA-PS as a Truthful Double Spectrum Auction with transmitting Primary users Sharing. We prove that TDSA-PS is truthful, individually rational, budget-balanced, and computationally efficient.**

1. Introduction

The cause for the scarcity of spectrum channels are two-folded: one resides in the limited bandwidth resource and the other resides in the exploding demand from spectrum-based services and devices. To help ease such scarcity, the concept of *cognitive radio networks* (CRNs) [11] is proposed, where licensed spectrum holders, a.k.a. *primary users* (PUs), may lease their licensed channels to users with no licensed spectrum channels, a.k.a. *secondary users* (SUs).

To further utilize the spectrum resource, users may share the same channel if their transmissions do not fail each other, which is referred to as *spatial reuse*. The depiction of such disruption lies in the interference model of the spectrum network. There are two commonly adopted interference models: *protocol model* and *physical interference model* [6]. In protocol model, two transmissions are considered to conflict with each other if the distance between the two users is within a given interference range. It determines the conflict of a transmission by considering the distance between two users, where in reality the interference to a transmission may be accumulated from multiple nearby transmissions. In this paper, we adopt physical interference model (a.k.a. SINR model), where a transmission is considered to be successful if its SINR ratio is above a given threshold. We will describe this model in details in Section 3-A.

CRN makes it possible for PUs to share their channels to SUs from a technology perspective. However, rational PUs may not voluntarily lease their channels to SUs without proper incentives to compensate their costs of maintaining and sharing the channels. Thus, incentive mechanisms for CRNs are designed accordingly where each SU would offer a price to purchase the channel for short term lease and each PU who shares its channel would receive a payment as a compensation. Auctions [9] are commonly adopted incentive mechanisms to properly allocate channels and compute payments for all PUs and SUs. *Single auctions* are the auctions that involve one seller and multiple buyers, and *double auctions* are the auctions that involve multiple sellers and multiple buyers. In CRN auctions, buyers are the SUs who purchase the channels for short terms and sellers are the PUs who lease the channels to SUs.

There are several important properties when designing effective and robust CRN auctions, such as truthfulness, individual rationality, budget-balance, and computational efficiency, the concepts of which will be presented in Section 3-C. Truthfulness is a very important economic property, as it ensures that each individual would reveal its valuation/cost honestly in order to maximize its own utility. Without truthfulness, dishonest users may increase their utilities strategically, leaving honest users in fear that the market price may be manipulated.

Most existing truthful CRN auctions assume that a transmitting PU does not lease its channel to SUs; if it leases its channel to SUs, it does not transmit itself [4, 5, 13, 14, 16, 18, 21, 22, 26, 27]. In practice, it is not compulsory that PUs cannot transmit while leasing the channels to SUs. Instead, when the transmissions of PUs and SUs do not disrupt one another, allowing transmitting PUs to share their channels to SUs would increase the amount of available spectrum resource for SUs to further ease the scarcity. Unfortunately, we cannot simple apply existing truthful CRN auctions to cope with this scenario. A major reason is that we need to consider the interference from SUs to PUs (whose transmissions should be guaranteed and not interrupted) on each channel, which complicates the channel allocation of the CRN under the SINR model.

In [24], a truthful single auction was proposed by Yang *et al.* which allows the transmitting PU to lease its channels to secondary users if the transmissions of the SUs do not undermine the transmission of the primary user. However, it assumes that there is only one PU whereas in practice, channels may come

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from multiple PUs. This prevents the study to be directly applied to the multi-PU scenario by neglecting the potential competitions among the PUs in the auction, which require more complicated channel allocation schemes and payment computation mechanisms. To cope with the multiple PUs scenario, we design **TDSA-PS** as a **T**ruthful **D**ouble **S**pectrum **A**uction with transmitting **P**rimary users **S**haring.

The main contributions of this paper are:

- To the best of our knowledge, we are the first to consider truthful double auctions for CRN where transmitting PUs also share the channels with SUs under the SINR model.
- We propose **TDSA-PS** to allocate channels and compute payments for all users, where transmitting PUs share their channels without undermining their own transmissions.
- We prove that **TDSA-PS** is truthful, individually rational, budget-balanced, and computationally efficient.

The remainder of this paper is organized as follows. In Section 2, we briefly review some state-of-art truthful CRN auctions. In Section 3, we present the CRN model and the auction model, introduce the desired properties, and state the design goal. In Sections 4 and 5, we propose **TDSA-PS** and present its analysis, respectively. We present our evaluation results and the corresponding analysis in Section 6 and draw our conclusions in Section 7.

2. Related Work

For truthful CRN single auctions, VERITAS [26] was proposed to achieve spatial reuse. Jia *et al.* proposed a revenue-maximization auction given prior knowledge on the distribution of the bids [8]. An $O(1)$-approximation algorithm was proposed to approximate revenue-maximization [1]. SMALL was designed [18] to allow each SU to request multiple channels. To further utilize the spectrum and improve the user satisfaction, SHIELD [16] was proposed. A truthful and revenue-maximization auction was proposed by Gopinathan and Li without prior knowledge on the bids [5]. Group-buying based auctions [19, 25] are proposed to enhance the buying power from SUs. ALETHEIA [12] achieves sybil-proofness, where users cannot benefit from making fake bids. SPECIAL was designed by Wu *et al.* [17] with adaptive-width channel allocation. Huang *et al.* designed a truthful spectrum sharing auction [7] which allocates the spectrum to a group of heterogenous users. Yi and Cai designed an ascending price auction [23] with power-constrained multi-radio SUs. Zhang *et al.* [24] proposed a truthful single auction where the PU and SUs may share the channels together. However, all of these auctions assume there is only one PU, whereas in practice there are often multiple PUs, which makes the competitions among PUs non-negligible.

For truthful CRN double auctions, Zhou *et al.* proposed TRUST with spatial reuse and homogenous channels [27]. Following this line, TAHES [4] extends the scenario into heterogenous channels. TAMES [2] further extends to where each SU may request multiple channels. DOTA [13] studies multi-channel scenario from PUs and SUs. Wang *et al.* proposed an online truthful double auction TODA [14] where users arrive in an online manner. Xu *et al.* studied a secondary spectrum market with asymptotic economic efficiency. Yang *et al.* proposed PROMISE [21] to approximate the platform utility. Wang *et al.* [15] studied truthful spectrum auctions with locality. A predictive double auction was proposed by Liu *et al.* [10]. None of these auctions considers the transmitting PUs sharing and cannot be directly applied to this scenario, as the allocation becomes more complicated with the interference from each PU.

3. System Model and Problem Formulation

We first present the CRN model and auction model. Then we introduce the desired properties and state the design goal.

A. CRN Model

There are m *primary users* $\mathcal{PU} = \{PU_1, ..., PU_m\}$. Each PU_i owns one licensed *channel* CH_i, where the channels are orthogonal. Let $\mathcal{C}$ denote the set of channels $\{CH_1, CH_2, ..., CH_m\}$. PU_i is equipped with a *transmitter* T_i to send signals with a fixed *transmission power* ρ_i. When leasing its channel CH_i to the SUs, PU_i transmits its own signal using CH_i at the same time. For each PU_i, it needs to transmit its signals to h_i *locations*: $\mathcal{L}_i = \{L_i^1, L_i^2, ..., L_i^{h_i}\}$.

There are n *secondary users* $\mathcal{SU} = \{SU_1, SU_2, ..., SU_n\}$. Each SU_j consists of a transmitter and a receiver: Γ_j and Υ_j. The transmission power of Γ_j is fixed at ϱ_j. SU_j requests d_j channels to transmit its signal and it does not differentiate the channels from one another. When it is clear from the context, we also use Γ_j and Υ_j to represent the locations of the transmitter and receiver, respectively.

Let y_i^j be the indicator that SU_j is allocated to transmit its signal in channel CH_i. If SU_j is assigned to CH_i, $y_i^j = 1$; $y_i^j = 0$ otherwise. SU_j's acquisition of d_j channels is indicated by $\sum_{i=1}^m y_i^j = d_j$. Define $\mathbf{y_i} = (y_i^1, y_i^2, ..., y_i^n)$ and $\mathbf{y} = (\mathbf{y_1}, \mathbf{y_2}, ..., \mathbf{y_m})$. Let $E(l_1, l_2)$ be the Euclidean distance from location l_1 to location l_2.

To measure if a transmission is *successful* at location L_i^l for PU_i, we use a concept named *Interference Temperature Limit* (ITL) [3], which is imposed by the Federal Communications Commission (FCC). If the cumulative interference received at a location is below its maximum tolerated ITL, the transmission is considered to be *successful*; and failed otherwise. Let $\gamma_i > 0$ be the maximum tolerated ITL for PU_i. PU_i's transmission at location L_i^l is successful if the following inequality holds:

$$\sum_{y_i^j=1} \frac{\varrho_j}{E(\Gamma_j, L_i^l)^\alpha} \leq \gamma_i, \tag{3.1}$$

where $\alpha \in [2, 4]$ is the path loss exponent [6]. The inequality indicates that the cumulated interference from the SUs who transmit signals using CH_i is below the maximum tolerated ITL. To make PU_i's transmission successful, we need to guarantee the inequality (3.1) to hold at each location $L_i^l \in \mathcal{L}_i$.

For each SU_j, its *Signal to Noise and Interference Ratio* (SINR) [6] in channel CH_i (if allocated) is defined as

$$SINR_i^j = \frac{\frac{\varrho_j}{E(\Gamma_j, \Upsilon_j)^\alpha}}{\frac{\rho_i}{E(T_i, \Upsilon_j)^\alpha} + N_0 + \sum_{j' \neq j, y_i^{j'}=1} \frac{\varrho_{j'}}{E(\Gamma_{j'}, \Upsilon_j)^\alpha}}, \tag{3.2}$$

where N_0 is the background noise. The numerator depicts the signal strength that is received at receiver R_j from transmitter T_j. The denominator consists of three parts: $\frac{\rho_i}{E(T_i,\Upsilon_j)^\alpha}$ depicts the interference at receiver R_j from primary user P_i, N_0 is the background noise, and $\sum_{j'\neq j, y_i^{j'}=1} \frac{\varrho_{j'}}{E(\Gamma_{j'},\Upsilon_j)^\alpha}$ is the cumulated interference from all SUs who are also assigned to use the same channel CH_i to transmit their signals. SU_j's transmission is considered to be *successful* if its $SINR$ is above a pre-defined $SINR$ threshold β_j for each of its assigned channels, i.e.,

$$SINR_i^j \geq \beta_j, \forall y_i^j = 1. \tag{3.3}$$

Feasible allocation: An allocation $\mathbf{y}$ is *feasible iff* the inequality (3.1) holds for each location L_i^l and the inequality (3.3) holds for each SU_j.

B. Auction Model

There is an *auctioneer* who conducts the double auction (e.g., the FCC). Each PU_i has a private *cost* $c_i > 0$ to manage leasing CH_i to the SUs. PU_i submits an *ask* $a_i > 0$ as the minimum required compensation for leasing CH_i. Note that a_i and c_i are not necessarily the same, since PU_i may benefit by strategically reporting $a_i \neq c_i$. We use the *ask vector* $\mathbf{a}$ to denote $(a_1, a_2, ..., a_m)$. Each SU_j has a private *valuation* $v_j > 0$ if it acquires at least d_j channels. SU_j submits a *bid* (b_j, d_j), where b_j is the maximum amount that SU_j is willing to pay for d_j channels. b_j and v_j are not necessarily the same, as well. We use the *bid vector* $\mathbf{b}$ to denote $((b_1, d_1), (b_2, d_2), ..., (b_n, d_n))$. Let x_i be the indicator of whether PU_i is a winner. If PU_i wins the auction and leases CH_i to SUs, $x_i = 1$; $x_i = 0$ otherwise.

Upon receiving the asks and bids, the auctioneer computes the winning indicator vector $\mathbf{x} = (x_1, x_2, ..., x_m)$ and the allocation indicator $\mathbf{y}$. Let p_i be the payment made to PU_i and q_j be the payment collected from SU_j. We define the payment vectors $\mathbf{p}$ as $(p_1, p_2, ..., p_m)$ and $\mathbf{q}$ as $(q_1, q_2, ..., q_n)$. PU_i's utility μ_i is defined as

$$\mu_i = (p_i - c_i)x_i, \tag{3.4}$$

which is the payment received minus the incurred cost. SU_j's utility ν_j is defined as

$$\nu_j = \begin{cases} v_j - q_j, & \sum_{i=1}^m y_i^j \geq d_j, \\ 0, & \text{otherwise}, \end{cases} \tag{3.5}$$

which is the valuation minus the payment made. The utility of the auctioneer u is defined as

$$u = \sum_{j=1}^n q_j - \sum_{i=1}^m p_i, \tag{3.6}$$

which is the difference between the payments collected from the SUs and the payments made to the PUs.

C. Desired Economic Properties and Design Goal

There are several desired economic properties that a good auction mechanism should possess. We list them in the following with the corresponding definitions.

- **Truthfulness:** An auction is truthful if for each PU (SU), it would reveal its cost (valuation) to maximize its utility, regardless of the asks and bids from the other users.
- **Individual Rationality:** An auction is individually rational if for each PU (SU), its utility is non-negative if it reveals its cost (valuation).
- **Budget-balance:** An auction is budget-balanced if the auctioneer's utility is non-negative.
- **Computational Efficiency:** An auction is computationally efficient if the mechanism can be conducted within a polynomial time complexity.

The goal of this paper is to design a spectrum double auction such that under the network and auction model presented in Section 3-A, it produces a feasible allocation $\mathbf{y}$ and payments $\mathbf{p}$ and $\mathbf{q}$ while being truthful, individually rational, budget-balanced, and computationally efficient.

4. Detailed Design of **TDSA-PS**

The rationale behind **TDSA-PS** is that we first sort all PUs in a non-decreasing order of asks and all SUs in a non-increasing order of per-channel bids. We try to find the smallest index k and the largest index l such that the first k PUs can be assigned to the first l SUs in a bid-independent allocation method, where the allocation is feasible. To guarantee truthfulness, we use the $k+1$-th PU's ask and $l+1$-th SU's per-channel bid to compute the payments for the winning users.

Algorithm 3 is the main algorithm for **TDSA-PS**. It calls two functions: $Avail()$ as in Algorithm 1, which returns a set of available channels for an SU; and $Allocate()$ as in Algorithm 2, which produces a feasible allocation to assign a subset of SUs to a subset of PUs. We present these two functions before the main algorithm of **TDSA-PS**.

Algorithm 1: $Avail(\mathbf{y}, \mathcal{PU}_i, j)$

```
1  C' ← ∅;
2  for i' ← 1, ..., m do
3      if PU_i' ∈ PU_i then
4          y^j_i' ← 1;
5          if the allocation y is feasible then
6              C' ← C' ∪ {CH_i'};
7          end
8          y^j_i' ← 0;
9      end
10 end
11 return C'
```

Algorithm 1 takes as input an allocation $\mathbf{y}$, a set of PUs $\mathcal{PU}_i$, and an index j of SU_j. It returns the set of channels $\mathcal{C}'$, which comes from the PU set $\mathcal{PU}_i$, such that if SU_j is assigned to channels in $\mathcal{C}'$, SU_j's transmissions are successful without failing the other transmissions allocated in $\mathbf{y}$.

First, the set $\mathcal{C}'$ is initialized to $\emptyset$ in Line 1, Algorithm 1. For each channel $CH_{i'}$ such that $PU_{i'} \in \mathcal{PU}_i$, we temporarily set the allocation $y_{i'}^j$ to 1 in Line 4, and check the feasibility of the allocation $\mathbf{y}$ in Line 5, where the feasibility definition of an allocation is introduced in Section 3-A. If the allocation is feasible, we add the channel $CH_{i'}$ into $\mathcal{C}'$ in Line 6 and reset the allocation $y_{i'}^{j}$ back to 0. The algorithm returns the set $\mathcal{C}'$.

Algorithm 2: $Allocate(k, l, \mathbf{a}, \mathbf{b}, \mathcal{PU}_k, \mathcal{SU}_l)$

1 $flag \leftarrow true$, $\mathbf{x}' \leftarrow \mathbf{0}$, $\mathbf{y}' \leftarrow \mathbf{0}$;
2 **for** $j' \leftarrow 1, ..., n$ **do**
3 **if** $SU_{j'} \in \mathcal{SU}_l$ **then**
4 $\mathcal{C}' \leftarrow Avail(\mathbf{y}, \mathcal{PU}_k, j')$;
5 **if** $|\mathcal{C}'| < d_{j'}$ **then**
6 $flag \leftarrow false$;
7 **else**
8 **for** $d' \leftarrow 1, ..., d_{j'}$ **do**
9 $k^* \leftarrow \arg\min_{k'}\{\sum_{g=1}^{m} {y'}_{k'}^{g} | CH_{k'} \in \mathcal{C}', {y'}_{k'}^{j'} = 0\}$;
10 ${x'}_{k^*} \leftarrow 1$, ${y'}_{k^*}^{j'} \leftarrow 1$;
11 **end**
12 **end**
13 **end**
14 **end**
15 **return** $(flag, \mathbf{x}', \mathbf{y}')$

Algorithm 2 assigns channels from PUs in $\mathcal{PU}_k$ to SUs in $\mathcal{SU}_l$. The algorithm takes as input the indexes k and l, the ask vector $\mathbf{a}$ and bid vector $\mathbf{b}$, the PU set $\mathcal{PU}_k$, and the SU set $\mathcal{SU}_l$. $Allocate$ returns the boolean indicator $flag$, the winning indicator $\mathbf{x}'$, and the allocation vector $\mathbf{y}'$, where $flag$ is set to $true$ if there is a feasible allocation, and $false$ otherwise.

In Line 1, $flag$ is set to $true$ initially, and the winning indicator $\mathbf{x}'$ and allocation vector $\mathbf{y}'$ are set to $\mathbf{0}$. From Line 2 to Line 14, we proceed with each $SU_{j'} \in \mathcal{SU}_l$. We use function $Avail()$ to compute the available channel set $\mathcal{C}'$ for $SU_{j'}$ in Line 4. If there are not enough channels to satisfy $SU_{j'}$'s demand $d_{j'}$ in Line 5, $flag$ is set to $false$. Otherwise, for each of the $d_{j'}$ channels allocated to $SU_{j'}$ (denoted as CH_{k*}), it is selected as the channel with the least number of SUs assigned in $\mathcal{C}'$ (Line 9). ${x'}_{k^*}$ and ${y'}_{k^*}^{j'}$ are updated accordingly in Line 10.

Algorithm 3 presents the main algorithm of **TDSA-PS**, which outputs the winning indicator vector $\mathbf{x}$, the allocation vector $\mathbf{y}$, and the payment vectors $\mathbf{p}$ and $\mathbf{q}$. We use an indicator $flag$ to denote if there is a feasible allocation.

In Line 1, the winning indicator $\mathbf{x}$, allocation vector $\mathbf{y}$, and payment vectors $\mathbf{p}$ and $\mathbf{q}$, are set to $\mathbf{0}$, initially. In Line 2 and Line 3, PUs are sorted in a non-decreasing order of their asks a_i and SUs are sorted in a non-increasing order of their per-channel bids $\frac{b_j}{d_j}$, respectively. l is set to $n-1$ and $flag$ is initialized to $false$ in Line 4. The outer "*while*" loop from Line 5 to Line 16, which allocates the channels for the first l SUs, stops at the condition that $flag$ becomes $true$ or l is less than 1. At the start of each iteration of the loops (Line 6), $\mathcal{SU}_l$ is defined as the first l SUs with the largest l per-channel bids. The number of PUs (channels) whom SUs in $\mathcal{SU}_l$ are assigned to, k, is initialized to 1. We use the inner "*while*" loop from Line 7 to Line 12 to compute the value of k for each l, where the loop continues if $flag = false$ (meaning no feasible allocation reached by far) and $k < m$. In each iteration of the inner loops, $\mathcal{PU}_k$ is defined as the first k PUs with the smallest k asks according to Line 8. Then we use the function $Allocate$ to see if there is a feasible allocation to assign SUs in $\mathcal{SU}_l$ to channels from $\mathcal{PU}_k$ in Line 9. To maintain the non-negative utility for the auctioneer according to its definition in Eq. (3.6), we compare the payment made to $\mathcal{PU}_k$ ($k \times a_{i_{k+1}}$) and the payment collected from $\mathcal{SU}_l$ ($\sum_{l'=1}^{l} d_{l'} \frac{b_{j_{l+1}}}{d_{j_{l+1}}}$) in Line 10. If the non-negative utility for the auctioneer cannot be guaranteed, $flag$ is set to $false$ in Line 11. If by the end of each inner "while" loop, no feasible allocation is found, k is increased by 1 (Line 13). If by the end of each outer "while" loop, no feasible allocation is found, l is decreased by 1 (Line 15). If there is a feasible allocation (indicated by $flag = true$ in Line 17), the payment for each winning PU is the $k+1$-th smallest ask $a_{i_{k+1}}$ (Line 18) and the per-channel payment for each winning $SU_{l'}$ is the $l+1$-th largest per-channel bid $\frac{b_{j_{l+1}}}{d_{j_{l+1}}}$, which makes the payment for $SU_{l'}$ be $d_{l'} \times \frac{b_{j_{l+1}}}{d_{j_{l+1}}}$ (Line 19).

Algorithm 3: TDSA − PS

1 $\mathbf{x} \leftarrow \mathbf{0}$, $\mathbf{y} \leftarrow \mathbf{0}$, $\mathbf{p} \leftarrow \mathbf{0}$, $\mathbf{q} \leftarrow \mathbf{0}$;
2 Sort PUs in non-decreasing order of a_i: $PU_{i_1}, PU_{i_2}, ..., PU_{i_m}$;
3 Sort SUs in non-increasing order of $\frac{b_j}{d_j}$: $SU_{j_1}, SU_{j_2}, ..., SU_{j_n}$;
4 $l \leftarrow n-1$, $flag \leftarrow false$;
5 **while** $flag = false$ *and* $l \geq 1$ **do**
6 $\mathcal{SU}_l \leftarrow \{SU_{i_1}, SU_{i_2}, ..., SU_{i_l}\}$, $k \leftarrow 1$;
7 **while** $flag = false$ *and* $k < m$ **do**
8 $\mathcal{PU}_k \leftarrow \{PU_{i_1}, PU_{i_2}, ..., PU_{i_k}\}$;
9 $(flag, \mathbf{x}, \mathbf{y}) \leftarrow Allocate(k, l, \mathbf{a}, \mathbf{b}, \mathcal{PU}_k, \mathcal{SU}_l)$;
10 **if** $k \times a_{i_{k+1}} \leq \sum_{l'=1}^{l} d_{l'} \frac{b_{j_{l+1}}}{d_{j_{l+1}}}$ **then**
11 $flag \leftarrow true$;
12 **end**
13 $k \leftarrow k+1$;
14 **end**
15 $l \leftarrow l-1$;
16 **end**
17 **if** $flag = true$ **then**
18 $p_{k'} \leftarrow a_{i_{k+1}}$, $\forall\ PU_{k'} \in \mathcal{PU}_k$;
19 $q_{l'} \leftarrow d_{l'} \frac{b_{j_{l+1}}}{d_{j_{l+1}}}$, $\forall\ SU_{l'} \in \mathcal{SU}_l$;
20 **end**
21 **return** $(\mathbf{x}, \mathbf{y}, \mathbf{p}, \mathbf{q})$

5. Economic Properties of TDSA-PS

Theorem 1: TDSA-PS is individually rational, budget-balanced, truthful, and computationally efficient. □

To prove Theorem 1, we prove the following lemmas.

Lemma 5.1: TDSA-PS is individually rational. □

Proof: Let PU_i reveal its cost in the auction. Suppose that it wins and its payment is $a_{i_{k+1}}$ in Line 18 of Algorithm 3. According to Line 8 of Algorithm 3, PU_i's ask value is lower than $a_{i_{k+1}}$. Thus, PU_i's utility is $\mu_i = p_i - c_i = a_{i_{k+1}} - a_i \geq 0$ by Eq. (3.4). If PU_i loses, its utility is 0 by Eq. (3.4).

Let SU_j reveal its valuation in the auction. Suppose that it wins and its payment is computed as $d_j \frac{b_{j'}}{d_{j'}}$ in Line 19 of Algorithm 3. Since SU_j is a winner, we have $\frac{b_j}{d_j} \geq \frac{b_{j'}}{d_{j'}}$ according to Line 6 of Algorithm 3. Line 8 of Algorithm 2 indicates that it is allocated with d_j channels. Thus, SU_j's utility is $\nu_j = v_j - q_j = b_j - d_j \frac{b_{j'}}{d_{j'}} \geq 0$ by Eq. (3.5). If SU_j loses, its utility is 0 by Eq. (3.5). ■

Lemma 5.2: TDSA-PS is budget-balanced. □

Proof: Line 10 of Algorithm 3 indicates that if the auction produces any winners, $k \times a_{i_{k+1}} \leq \sum_{l'=1}^{l} b_{j_{l+1}} \frac{d_{l'}}{d_{j_{l+1}}}$. Since $a_{i_{k+1}}$ is the payment for each winning PU_i, and $d_{l'} \frac{b_{j_{l+1}}}{d_{j_{l+1}}}$ is the payment for each winning $SU_{l'}$ (Lines 18 and 19 of Algorithm 3), we have $u = \sum_{j=1}^{n} q_j - \sum_{i=1}^{m} p_i \geq 0$. If there is no winner, the auctioneer's utility is 0 by Eq. (3.6). ■

Lemma 5.3: TDSA-PS is truthful. □

Proof: For each PU_i, if it is a winner by revealing $a_i = c_i$, suppose that the winning PU set is $\mathcal{PU}_{k^*}$. PU_i's utility is $a_{i_{k^*+1}} - c_i \geq 0$ according to Lemma 5.1. If by changing its ask it remains a winner among the top k^* PUs, there is still a feasible allocation for these winners since the assignment in Algorithm 2 is bid-independent. Thus, with the same payment $a_{i_{k^*+1}}$, and PU_i's utility does not change. If PU_i loses the auction, its utility drops to 0.

Now suppose that PU_i is a loser by revealing $a_i = c_i$, we have $a_i \geq a_{i_{k^*+1}}$ and PU_i's utility is 0. If by changing its ask, it becomes a winner, the new winning PU set $\mathcal{PU}_{k'}$ must satisfy $k' \leq k^*$. This is because when $a_i = c_i$, $Allocate$ returns $true$ at $\mathcal{PU}_{k^*}$ and returns false at $\mathcal{PU}_{k^*+1}$. After PU_i changes its position, $Allocate$ returns $true$ no later than it reaches to $\mathcal{PU}_{k^*}$. Thus, the new payment is $a_{i_{k'+1}} \leq a_{i_{k^*+1}} \leq a_i$, which makes PU_i's utility non-positive. If by changing its ask, PU_i remains a loser, its utility remains 0.

For each SU_j, suppose that it is a winner by revealing $b_j = v_j$ and the winning SU set is $\mathcal{SU}_{l^*}$. Its utility is $v_j - d_j \frac{b_{j_{l^*+1}}}{d_{j_{l^*+1}}} \geq 0$ according to Lemma 5.1. If by changing its bid, it remains a winner among the top l^* SUs, there is still a feasible allocation for these winners since the assignment in Algorithm 2 is bid-independent. Thus, the payment is $d_j \frac{b_{j_{l'+1}}}{d_{j_{l'+1}}} \geq d_j \frac{b_{j_{l^*+1}}}{d_{j_{l^*+1}}}$, which does not increase SU_j's utility. SU_j cannot be a winner if it does not belong to $\mathcal{SU}_{j^*}$ because adding SUs into $\mathcal{SU}_{l^*}$ makes $Allocate$ returns $false$ in the iteration for $\mathcal{SU}_{l^*+1}$. If by changing its bid SU_j becomes a loser, its utility is 0.

Now suppose that SU_j is a loser by revealing $b_j = v_j$, and the winning SU set is $\mathcal{SU}_{l^*}$. We have $\frac{v_j}{d_j} \leq \frac{b_{j_{l^*+1}}}{d_{j_{l^*+1}}}$. If by changing its bid, SU_j remains a loser, its utility remains 0. If by changing its bid, SU_j becomes a winner in the winning SU set $\mathcal{SU}_{l'}$, we have $l' \leq l^*$. This is because there is no feasible allocation for $\mathcal{SU}_{l^*+1}$ when $b_j = v_j$. In the new allocation, to put SU_j as a winner, at least one of the SUs in the original $\mathcal{SU}_{l^*}$ becomes a loser to make room for SU_j, which makes $l' \leq l^*$. Therefore, SU_j's new utility is $v_j - d_j \frac{b_{j_{l'+1}}}{d_{j_{l'+1}}} \leq v_j - d_j \frac{b_{j_{l^*+1}}}{d_{j_{l^*+1}}} \leq 0$. ■

Lemma 5.4: TDSA-PS is computationally efficient. □

Proof: The time complexity of **TDSA-PS** is $O(m^2 n^2 (mn + n^2 + nL))$, where $L = \max\{|\mathcal{L}_i|\}$. ■

With Lemmas 5.1 to 5.4, Theorem 1 is proved.

6. Performance Evaluation

We evaluate the performance of **TDSA-PS**. We compare the results of **TDSA-PS** with those of DOTA [13], as the two mechanisms both study truthful CRN double auctions where SUs request multiple channels. To make fair comparisons, we apply the group formation results of **TDSA-PS** to DOTA. Implementations were run on a Linux system with Intel Core I7-4700 3.5Hz and 16GB memory.

In DOTA, after the groups of SUs are given, a group bid is computed as the minimum bid of the SU in the group and the group demand is the largest demand of the SU in the group. Then SU groups are assigned to channels in favor of the group bid. Each winning PU's payment equals to the smallest losing PU's ask, and each winning SU group's payment equals to the largest losing SU group bid. In each winning SU group, all SUs share the group payment.

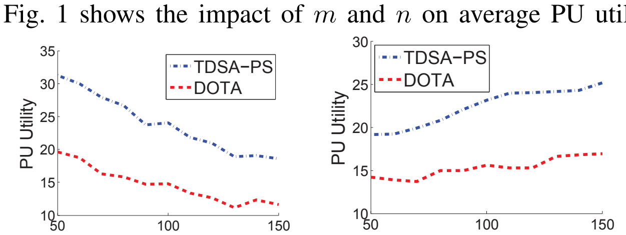
A. Environment Setup and Performance Metrics

Environment Setup: Most of the CRN parameters in this paper are the same as in [20]. The geographic area of the CRN is defined as a square of size 1000×1000. All transmitters and receivers are distributed uniformly at random within. For each PU_i, the number of transmitting destination $|\mathcal{L}_i|$ is randomly distributed over $[1, 5]$. The powers of all transmitters are set uniformly at $200w$. All asks and per-channel bids are distributed uniformly at random over $(0, 100]$. We set the path loss exponent $\alpha = 3.5$, the tolerated ITL $\gamma_i = 10$ for all PUs, and the SINR threshold $\beta_j = 10$ for all SUs. The background noise N_0 is set to 10^{-9}.

Performance Metrics: We choose *average PU utility*, *average SU utility*, and *auctioneer's utility* as the performance metrics. For each metric, we evaluate the impact from the size of the PUs and SUs (m and n), by setting $m = 50$, $n \in [50, 150]$ and $n = 50$, $m \in [50, 150]$, respectively. All results are averaged over 100 times for each parameter configuration.

B. Evaluation Results and Analysis

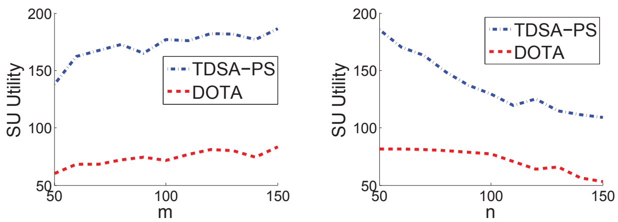
Fig. 1 shows the impact of m and n on average PU utility.



(a) $50 \leq m \leq 150$ (b) $50 \leq n \leq 150$

Fig. 1. Average PU Utility

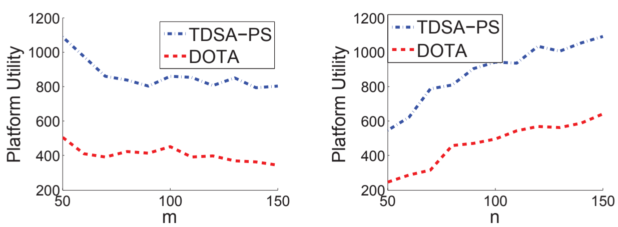
From Fig. 1(a), we observe that the average PU utility decreases with the increment of m. as with more PUs, the competition among PUs becomes more fierce. From Fig. 1(b), we observe that the average PU utility increases with the increment of n. This is because with more SUs to purchase the channels, the payments made to the PUs increase. Moreover, the average PU utility from **TDSA-PS** is higher than that of DOTA, as **TDSA-PS** charges each PU the $k+1$-th lowest ask, and DOTA charges each PU the $\min\{m^*, n^*\}$-th lowest price, where m^* is the number of winning PUs and n^* is the number of winning SU groups. In most cases, $k+1 < \min\{m^*, n^*\}$.



(a) $50 \le m \le 150$ (b) $50 \le n \le 150$

Fig. 2. Average SU Utility

Fig. 2 shows the impact of m and n on average SU utility. From Fig. 2(a), we observe that the average SU utility increases with the increment of m where as from Fig. 2(b), we observe that it decreases with the increment of n. The reasons are "symmetric" to those for Fig. 1: more PUs results in higher payments for each winning SU; more SUs introduce more fierce competition among the SUs. We also observe that the average SU utility of **TDSA-PS** is higher than that of DOTA, as **TDSA-PS** pays each SU the $l+1$-th highest per-channel bid and in DOTA SUs share payments from the largest losing SU group.



(a) $50 \le m \le 150$ (b) $50 \le n \le 150$

Fig. 3. Platform Utility

In Fig. 3, we monitor the impact of m and n on the auctioneer's utility. From Fig. 3(a), we observe that the auctioneer's utility decreases slowly with the increment of m. This is because with more winning PUs, the total payment paid to the PUs increases. From Fig. 3(b), we observe that the auctioneer's utility increases with the number of n, as with more SUs, the total payment collected from the SUs increases. Moreover, the auctioneer's utility from **TDSA-PS** is higher than that of DOTA. This is because in **TDSA-PS**, there are more winning SUs to provide a higher payment made to the auctioneer.

7. Conclusions

We study truthful double auction design in CRNs where transmitting PUs may lease their channels to SUs. We have proposed **TDSA-PS** and proved that it is truthful, individually rational, budget-balanced, and computationally efficient. Extensive evaluation results are presented and analyzed.

References

[1] M. Al-Ayyoub and H. Gupta, "Truthful spectrum auctions with approximate revenue," in *IEEE INFOCOM'11*, pp. 2813 – 2821.

[2] Y. Chen, J. Zhang, K. Wu, and Q. Zhang, "TAMES: A truthful double auction for multi-demand heterogeneous spectrums," *IEEE TPDS*, vol. 25, pp. 3012–3024, 2014.

[3] F. C. Commission, "Estabilishment of interference temperature metric to quantify and mangage interference and to expand available unlicensed operation in certain fixed mobile and satellite frequency bands," *Et Docket*, no. 03-237, 2003.

[4] X. Feng, Y. Chen, J. Zhang, Q. Zhang, and B. Li, "TAHES: Truthful double auction for heterogeneous spectrums," in *IEEE INFOCOM'12*, pp. 3076–3080.

[5] A. Gopinathan and Z. Li, "A prior-free revenue maximizing auction for secondary spectrum access," in *IEEE INFOCOM'11*, pp. 86–90.

[6] P. Gupta and P. R. Kumar, "The capacity of wireless networks," *IEEE TIT*, vol. 46, pp. 388–404, 2000.

[7] J. Huang, R. A. Berry, and M. L. Honig, "Auction-based spectrum sharing," *Mob. Netw. Appl.*, vol. 11, pp. 405–418, 2006.

[8] J. Jia, Q. Zhang, Q. Zhang, and M. Liu, "Revenue generation for truthful spectrum auction in dynamic spectrum access," in *MobiHoc'09*, pp. 3–12.

[9] V. Krishna, in *Auction Theory*. Academic Press, 2010.

[10] Z. Liu and C. Li, "On spectrum allocation in cognitive radio networks: a double auction-based methodology," *Wireless Networks*, vol. 23, pp. 453–466, 2017.

[11] V. Valenta, R. Marlek, G. Baudoin, M. Villegas, M. Suarez, and F. Robert, "Survey on spectrum utilization in europe: Measurements, analyses and observations," in *CROWNCOM'10*, pp. 1–5.

[12] Q. Wang, B. Ye, B. Tang, T. Xu, S. Guo, S. Lu, and W. Zhuang, "Robust large-scale spectrum auctions against false-name bids," *IEEE TMC*, vol. 16, pp. 1730–1743, 2017.

[13] Q. Wang, B. Ye, T. Xu, S. Lu, and S. Guo, "DOTA: A double truthful auction for spectrum allocation in dynamic spectrum access," in *IEEE WCNC'12*, pp. 1490–1495.

[14] S. Wang, P. Xu, X. Xu, S. Tang, X. Li, and X. Liu, "TODA: Truthful online double auction for spectrum allocation in wireless networks," in *IEEE DySPAN'10*, pp. 1–10.

[15] W. Wang, B. Liang, and B. Li, "Designing truthful spectrum double auctions with local markets," *IEEE TMC*, vol. 13, pp. 75–88, 2014.

[16] Z. Wei, T. Zhang, F. Wu, G. Chen, and X. Gao, "SHIELD: A strategy-proof and highly efficient channel auction mechanism for multi-radio wireless networks," in *WASA'12*, pp. 72–87.

[17] F. Wu, T. Zhang, C. Qiao, and G. Chen, "A strategy-proof auction mechanism for adaptive-width channel allocation in wireless networks," *IEEE JSAC*, vol. 34, pp. 2678–2689, 2016.

[18] F. Wu and N. Vaidya, "SMALL: A strategy-proof mechanism for radio spectrum allocation," in *IEEE INFOCOM'11*, pp. 81 – 85.

[19] D. Yang, G. Xue, and X. Zhang, "Group buying spectrum auctions in cognitive radio networks," *IEEE TVT*, vol. 66, pp. 810–817, 2017.

[20] D. Yang, X. Fang, N. Li, and G. Xue, "A simple greedy algorithm for link scheduling with the physical interference model," in *IEEE GLOBECOM'09*, pp. 1–6.

[21] D. Yang, X. Zhang, and G. Xue, "PROMISE: a framework for truthful and profit maximizing spectrum double auctions," in *IEEE INFOCOM'14*, pp. 109–117.

[22] E. Yao, L. Lu, and W. Jiang, "An efficient truthful double spectrum auction design for dynamic spectrum access," in *CROWNCOM'11*, pp. 181–185.

[23] C. Yi and J. Cai, "Ascending-price progressive spectrum auction for cognitive radio networks with power-constrained multi-radio secondary users," *IEEE TVT*, vol. PP, 2017.

[24] Y. Zhang, D. Yang, G. Xue, and J. Tang, "A spectrum auction under physical interference model," in *IEEE GLOBECOM'16*.

[25] F. Zhao, H. Nie, and H. Chen, "Group buying spectrum auction algorithm for fractional frequency reuse cognitive cellular systems," *Ad Hoc Networks*, vol. 58, pp. 239 – 246, 2017.

[26] X. Zhou, S. Gandhi, S. Suri, and H. Zheng, "eBay in the sky: Strategy-proof wireless spectrum auctions," in *ACM MOBICOM'08*, pp. 2–13.

[27] X. Zhou and H. Zheng, "TRUST: A general framework for truthful double spectrum auctions," in *IEEE INFOCOM'09*, pp. 999–1007.