A Spectrum Auction under Physical Interference Model

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***Abstract*—Spectrum auctions provide a platform for licensed spectrum users to share their underutilized spectrum with unlicensed users. Existing spectrum auctions either use the protocol interference model to characterize interference relationship as binary relationship, or do not allow the primary and secondary users to share channels simultaneously. To fill this void, we design SPA, a spectrum single-sided auction under the physical interference model, which considers the interference to be accumulative. We prove that SPA is truthful, individually rational, and computationally efficient. Results from extensive simulation studies demonstrate that, SPA achieves higher spectrum utilization and buyer satisfaction ratio, compared with an existing auction adapted for the physical interference model.**

I. Introduction

Spectrum is a critical yet scarce resource due to the substantial growth of wireless technology and applications. Indeed, Federal Communications Commission (FCC) and its counterparts across the world have released licenses of unused spectrum and collected billions of dollars in the past decade.

Fundamentally different from conventional goods, spectrum is reusable, which is referred to as *spatial reusability*. Users can share the same channel as long as they can transmit signals simultaneously without disrupting each other's transmission. Therefore, the primary license holders may be motivated to open up their underutilized spectrum for sharing, so that they may make profit by leasing access to spectrum resources. In addition, allowing spectrum to be shared by multiple users can also improve the spectrum utilization efficiency.

When the spatial reusability of the spectrum is considered, one arising challenge is to characterize interference relationship among users in cognitive radio networks (CRNs). Most of the existing spectrum auctions adopt the *protocol interference model* [6], which simplifies the step of allocation by scheduling users according to conflict graphs. The functionality of the mechanisms is the assumption that the interference relationship between any two users can be modeled based on the protocol. In other words, the interference relationship is binary. But in practice for wireless networks, a conflict graph may not be precise, as the interference from other users is accumulative.

To solve this problem, we intend to design spectrum auctions without using the given conflict graph, but under the *physical interference model* [6] instead. Next we shall explain the protocol and physical models in details.

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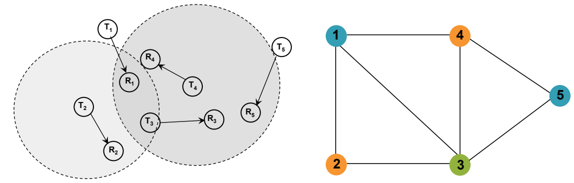
A. Interference Models

1) Protocol Interference Model [6]: When two users transmit using the same channel simultaneously, they interfere with each other if the distance between them is within interference range. Usually, a conflict graph is used to characterize the interference relationship under the protocol interference model, where each node represents a user, and an edge exists if two nodes interfere with each other. For example, Figure 1 shows a wireless network under the protocol interference model and the corresponding conflict graph. For clarity, we only show the interference ranges of Link (T_2, R_2) and Link (T_4, R_4) in Figure 1(a). Unfortunately, this simplified model abstracts away the accumulative nature of interference. Even if a single transmitter far away from a receiver may not corrupt the transmission, the accumulated interference from several such nodes could still generate enough interference to prevent the receiver from successfully decoding the received message.



(a) Wireless network (b) Conflict graph

Figure 1. Protocol interference model

2) Physical Interference Model [6]: The physical interference model (a.k.a. SINR model) computes the Signal to Interference and Noise Ratio (SINR) of each user and compares this value with a threshold. If the SINR value is no less than the threshold, the signal transmission is considered successful for the corresponding user, and it is considered unsuccessful otherwise. Figure 2 shows the same wireless network as in Figure 1, under the SINR model, where the interference is characterized as accumulated. For example, Link (T_1, R_1) receives interference from all other links. Compared with the protocol interference model, the physical interference model has been recognized as a more realistic model in wireless communications.

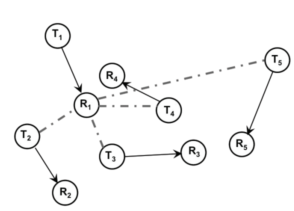


Figure 2. Physical interference model

The main contributions of this paper are:

- In this paper, to the best of our knowledge, we are the first to design a single-sided **SP**ectrum **A**uction, named SPA, with spatial reusability under the physical interference model. SPA consists of an allocation algorithm and a pricing mechanism.
- SPA allows the primary user and secondary users to share channels simultaneously, as long as the signal quality of the primary user is guaranteed.
- We rigorously prove that SPA is truthful, individually rational, and computationally efficient.

The remainder of the paper is organized as follows. In Section II, we give a brief review of related work in the literature. In Section III, we formally describe the CRN model as well as the auction model. We present our designed auction, SPA and analyze the properties of SPA in Section IV. We evaluate the performance of SPA by comparing it with an existing auction in Section V and conclude this paper in Section VI.

II. Related Work

As pioneers in spectrum auction design, Zhou *et al.* [21] proposed VERITAS under the protocol model, the first truthful auction considering the spectrum reusability and computation efficiency. In [8], based on the concept of virtual valuation, Jia *et al.* designed an exponential time VCG-based auction to maximize the expected revenue. Along this line, Al-Ayyoub and Gupta [1] designed a polynomial time spectrum auction that yields approximated expected revenue. In [15], Wu and Vaidya designed SMALL to guarantee that the owner's utility is non-negative in the scenario where the owner of the spectrum has a reserved price for each of the channels. Following the same design methodology, Wei *et al.* [14] designed SHIELD that improves spectrum utilization and buyer satisfaction compared with VERITAS and SMALL. Inspired by the group-buying service on the Internet, Lin *et al.* [10] designed a three-state auction, called TASG that allows a leader in each group to conduct an outer auction for aggregating the bids within the group. Along this line, Yang *et al.* [18] designed TRUBA that significantly increases the revenue. In [5], Gopinathan and Li studied spectrum auctions with prior-free setting and designed a truthful auction to approximately maximize the revenue.

TRUST [22] is the first truthful double auction designed for spectrum trading. Feng *et al.* [4] extended to heterogeneous spectrum auctions and designed TAHES. In [12], a double truthful auction, called DOTA, was proposed to allow each user to bid for more than one channel. Considering the fact that secondary users may join the network in an online fashion, Wang *et al.* [13] designed TODA. In [19], Yang *et al.* proposed PROMISE for maximizing the profit without the knowledge of the users valuation distribution.

In the scenario of the physical interference model, Kakhbod *et al.* in [9] developed a truthful auction for dividing a spectrum channel into several small channels with less bandwidth, where all transmitters power levels are fixed homogeneously. In [2], a truthful single auction was studied by Bae *et al.*, where a sequential auction (an auction with multiple rounds) was used to reach a pure strategy equilibrium. Huang *et al.* also introduced a truthful auction-based spectrum sharing mechanism [7] where a group of users compete for a spectrum channel under different definitions of their utilities. Zhang *et al.* proposed TSA [20], a framework for truthful double auctions under the physical interference model with power control. To the best of our knowledge, there is no truthful single-sided auction for spectrum sharing under the physical interference model.

III. System Model

In this section, we describe the necessary concepts in cognitive radio networks and the physical interference model for the spectrum auction.

A. Cognitive Radio Network Model

We consider a cognitive radio network (CRN) consisting of one primary user (PU) and a set $\mathcal{S} = \{S_1, S_2, \ldots, S_n\}$ of n secondary users (SUs). The PU, e.g., the TV broadcaster, owns m licensed channels $\mathcal{C} = \{c_1, c_2, \ldots, c_m\}$, and is willing to rent the spectrum for profit. The channels are assumed to be orthogonal, which means that there is no interference among users using different channels. Let P_0 denote the transmission power of the PU's transmitter, e.g., the TV tower, denoted by T_0. SUs do not have licensed spectrum channels, but are willing to pay for channels from the PU in the short term. Each $S_i \in \mathcal{S}$ is a transmitter-receiver pair (T_i, R_i). Let P_i denote the transmission power of T_i.

We allow the PU and SUs to transmit signals over the same channels simultaneously. Let $\mathcal{C}_0 \subseteq \mathcal{C}$ represent the channels that the PU is currently using. To protect the transmission of the PU from being interrupted by the transmissions of SUs, the FCC proposed a metric, named Interference Temperature Limit (ITL) [3], which sets the maximum cumulative amount of interference that can be tolerated at the certain locations. Let $\mathcal{L} = \{l_1, l_2, \ldots, l_h\}$ denote the locations where the PU measures ITL. We use γ_j to represent PU's tolerated ITL at location l_j. With this setting, the PU can lease its channels to SUs as long as the transmissions of the selected SUs do not cause more interference than γ_j, for any $l_j \in \mathcal{L}$. Assume G_k is the group of SUs assigned to the same channel c_k. Throughout the rest of this paper, we use channel and group interchangeably. The ITL constraints can be represented by

$$\mathbb{1}_{\mathcal{C}_0}(c_k) \sum_{S_i \in G_k} \frac{P_i}{d(T_i, l_j)^{\alpha}} \leq \gamma_j, \forall l_j \in \mathcal{L}, \tag{1}$$

where $\mathbb{1}_{\mathcal{C}_0}(c_k)$ is an indicator function defined as

$$\mathbb{1}_{\mathcal{C}_0}(c_k) = \begin{cases} 1, & c_k \in \mathcal{C}_0, \\ 0, & c_k \notin \mathcal{C}_0, \end{cases} \tag{2}$$

$d(T_i, l_j)$ is the maximum of 1 and the Euclidean distance from transmitter T_i to location l_j, and α is the path loss exponent with value between 2 and 4 usually.

We can achieve spatial reuse by assigning multiple SUs to the same channel, if they can transmit simultaneously while each obtains a satisfactory SINR value.

The Signal to Interference and Noise Ratio (SINR) [6] of S_i in G_k is:

$$\frac{\frac{P_i}{d(T_i,R_i)^\alpha}}{\mathbb{1}_{\mathcal{C}_0}(c_k)\frac{P_0}{d(T_0,R_i)^\alpha}+\sum_{S_i\neq S_j\in G_k}\frac{P_j}{d(T_j,R_i)^\alpha}+N_0}\geq\beta_i, \quad (3)$$

where N_0 is the ambient noise power level, and $\mathbb{1}_{\mathcal{C}_0}(c_k)$ is defined in Equation (2).

If Condition (3) is satisfied, the transmission is considered successful; otherwise, the transmission is considered unsuccessful. We assume that Condition (3) is satisfied for each S_i when it solely occupies a channel. We can preprocess this before our proposed mechanism, that if by transmitting at a power of P_i, Condition (3) is not satisfied, when S_i solely occupies a channel, we discard S_i out of the market since no channel could satisfy a successful transmission for S_i.

Before we formally describe our algorithm, we need the following definitions: *SU Tolerance* [16, 17] and *Feasible Group*.

Definition 1 (SU Tolerance)**.** *The tolerance τ_i indicates how much interference S_i can endure before the corresponding SINR value falls below the threshold β_i. It can be calculated by*

$$\tau_i=\frac{\frac{P_i}{d(T_i,R_i)^\alpha}}{\beta_i}-N_0. \quad (4)$$

Definition 2 (Feasible Group)**.** *A group G_k of SUs is feasible with respect to S_i if, after the addition of S_i to the group, Condition* (3) *is satisfied $\forall S_j\in G_k\cup\{S_i\}$ and Condition* (1) *is satisfied for the PU.*

B. Auction Model

With the primary and secondary users in the cognitive radio network, we aim to design a single-sided spectrum auction that is individually rational, computationally efficient, and truthful. In this setting, the PU is the seller and SUs are buyers. Throughout the rest of the paper, we use the terminology of PU and seller, SU and buyer interchangeably. The PU contributes m *homogeneous* channels $\{c_1,c_2,...,c_m\}$ and is using channels $c_k\in\mathcal{C}_0$. Each buyer S_i requests d_i channels and holds a private valuation $v_i\geq 0$ for leasing d_i channels, as well as a bid $b_i\geq 0$ as the maximum amount that it would pay for d_i channels. In this paper, we focus on the single-minded scenario: a buyer accepts either d_i channels or 0 channel. Another possible case is range-based: a buyer accepts any x_i channels if $0\leq x_i\leq d_i$. The auction design for the range-based case will be our future work.

The auction works as follows: after collecting the bids and requests from all buyers, the algorithm decides the allocation for each buyer. The algorithm also computes the payment for each winning buyer. Buyer S_i pays p_i as the corresponding payment.

The utility of S_i is defined as follows:

$$u_i=\begin{cases}v_i-p_i, & if\ S_i wins,\\ 0, & otherwise.\end{cases} \quad (5)$$

C. Desired Properties

There are three desired properties for an auction to satisfy:

- *Truthfulness*: an auction is truthful if a buyer bids the true valuation of the resource, its utility will not be less than that when it lies.
- *Individual Rationality*: an auction is individually rational if all buyers have non-negative utilities by revealing their true valuations.
- *Computational Efficiency*: an auction is computationally efficient if it can be conducted within polynomial time.

IV. Auction Design of SPA

In this section, we introduce the basic design of SPA, where buyers are assumed to be single-minded, i.e., a buyer S_i that requests d_i channels only accepts either all d_i channels or nothing.

A. High-level Description

SPA consists of two stages: allocation and pricing. The allocation stage applies a mechanism, which sorts buyers based on both bids and tolerances. Then we check the feasibility of each buyer to m channels sequentially, and assign the buyer to the first d_i feasible channels as a winner. In the pricing stage, we determine the final payment for each winner. The pricing stage applies a mechanism that aims to find critical values. We present the detailed algorithms in the following two subsections.

B. Channel Allocation

We start with an intuitive idea. When we choose a buyer from the set *S* to allocate a channel, the one with a higher per-channel bid and more tolerance is preferred. In other words, this buyer is willing to pay more for each channel and is more resistant to interference. This property is best characterized by the product:

$$\tilde{b}_i=\frac{b_i}{d_i}\cdot\tau_i. \quad (6)$$

Without loss of generality, we can sort all SUs based on $\tilde{b}_i$ in a non-increasing order $\tilde{b}_1\geq\tilde{b}_2\geq,\ldots,\geq\tilde{b}_n$ and get a sorted list $\mathbb{S}:\mathbb{S}_1,\mathbb{S}_2,\mathbb{S}_3,\ldots,\mathbb{S}_n$.

Based on the sorted list $\mathbb{S}$, Algorithm 1 allocates buyers sequentially from $\mathbb{S}_1$ to $\mathbb{S}_n$. For each buyer $\mathbb{S}_i$, the algorithm checks whether G_k is feasible to $\mathbb{S}_i$ for $k=1$ to m. We use a binary variable f_{ik} to mark the feasibility status for $\mathbb{S}_i$, defined as:

$$f_{ik}=\begin{cases}1, & \text{if } G_k \text{ is feasible to } \mathbb{S}_i,\\ 0, & \text{otherwise.}\end{cases} \quad (7)$$

The algorithm assigns $\mathbb{S}_i$ to the first d_i feasible channels as the buyer requests. We use another binary variable a_{ik} to mark the allocation status for $\mathbb{S}_i$. If G_k is allocated to $\mathbb{S}_i$, then $a_{ik}=1$; otherwise 0. If there are less than d_i feasible channels to $\mathbb{S}_i$, the algorithm assigns $\mathbb{S}_i$ nothing.

Algorithm 1: Allocation ($\mathbb{S}$)

```
for k ← 1 to m do G_k ← ∅;
for i ← 1 to n do
    for k ← 1 to m do Initialize f_ik using (7);
    if Σ_{k=1}^{m} f_ik ≥ d_i then
        for k ← 1 to m do
            if f_ik = 1 and Σ_{k=1}^{m} a_ik < d_i then
                a_ik ← 1; G_k ← G_k ∪ {S_i};
            end
        end
    end
end
𝒢 ← {G_1, G_2, ..., G_m};
return 𝒢
```

C. Pricing

In this stage, we compute payments for winners.

With each buyer either assigned d_i channels or nothing, next we need to compute their payments. To maintain truthfulness, we find each winning buyer its critical value [11].

Definition 3 (Critical Value)**.** *The critical value is the the smallest value such that a buyer will win when bidding higher than this value, and it will lose when bidding lower than that.*

Algorithm 2: Pricing($\mathbb{S}, \mathcal{G}$)

```
1  for i ← 1 to n do p_i ← 0;
2  𝒲 ← ⋃_{G_k ∈ 𝒢} G_k;
3  for S_i ∈ 𝒲 do
4      S^[-i] ← S \ {S_i};
5      for k ← 1 to m do
6          G_k ← ∅; Initialize f_ik using (7);
7      end
8      for q ← 1 to n − 1 do
9          for k ← 1 to m do Initialize f_qk using (7);
10         if Σ_{k=1}^{m} f_qk ≥ d_q then
11             for k ← 1 to m do
12                 if f_qk = 1 and Σ_{k=1}^{m} a_qk < d_q then
13                     a_qk ← 1; G_k ← G_k ∪ {S_q^[-i]};
14                     if G_k is infeasible to S_i then
15                         f_ik ← 0;
16                         if Σ_{k=1}^{m} f_ik < d_i then
17                             p_i ← d_i (b̃_q / τ_i); break;
18                         end
19                     end
20                 end
21             end
22         end
23         if p_i > 0 then break;
24     end
25 end
26 return {p_1, p_2, ..., p_n}
```

Algorithm 2 illustrates the payment computation for all winners. The basic idea is that for each winner $\mathbb{S}_i$, first take $\mathbb{S}_i$ out of the sorted list $\mathbb{S}$ and get a sorted list $\mathbb{S}^{[-i]}$ consisting of the remaining buyers. Then allocate channels to the remaining buyers. Each time when assigning a channel to a remaining buyer, check the feasibility of $\mathbb{S}_i$. When we find the first buyer $\mathbb{S}_q^{[-i]}$, who makes $\mathbb{S}_i$'s request unsatisfied, its corresponding $d_i\frac{\tilde{b}_q}{\tau_i}$ is $\mathbb{S}_i$'s critical value. Line 17 indicates the payment p_i for $\mathbb{S}_i$. If we cannot find the critical value for $\mathbb{S}_i$, then the payment is 0.

We shall run Algorithm 2 to compute the payments for all the buyers.

D. Analysis

We prove that SPA satisfies the desired properties introduced in Section I.

Theorem 1. *SPA is truthful, individually rational, and computationally efficient.*

We prove Theorem 1 by the following three lemmas.

Lemma 1. *SPA is truthful.*

Proof: It is known that an auction is truthful if the allocation algorithm of this auction is monotone while the price charged of a winner is its critical value [11].

Monotonic allocation: In the following, we prove that, for each buyer S_i, if S_i wins by bidding b_i, then it also wins by bidding $b'_i > b_i$.

Suppose S_i wins by bidding b_i. Let $\mathbb{S}$ and $\mathbb{S}'$ be the sorted lists when S_i bids b_i and b'_i, respectively. With $b'_i > b_i$, we have $\frac{b'_i}{d_i} > \frac{b_i}{d_i}$ and $\tilde{b}'_i > \tilde{b}_i$. Therefore S_i's position in $\mathbb{S}$ is after that in $\mathbb{S}'$ with the same τ_i and d_i. Because S_i wins by bidding b_i, there are at least d_i feasible channels for S_i when S_i is considered according to $\mathbb{S}$. It implies that there are also at least d_i feasible channels for S_i when S_i is considered according to $\mathbb{S}'$. Thus S_i wins by bidding b'_i as well. This proves that the allocation is monotonic.

Critical Value: In the following, we prove that, for each buyer S_i, its payment p_i is its critical value, i.e., S_i wins by bidding higher than p_i and loses by bidding lower than p_i.

We consider the following two cases separately:

- *Case 1: $b_i > p_i$*
  With $p_i = d_i\frac{\tilde{b}_q}{\tau_i}$, we have $\tilde{b}_i > \tilde{b}_q$ and S_i would be ranked before S_q in the sorted list. Because S_q is the first buyer who makes S_i have less than d_i feasible channels, being ranked before S_q guarantees that S_i has at last d_i feasible channels. Therefore, S_i wins.
- *Case 2: $b_i < p_i$*
  Similarly as the Case 1 above. S_i would be ranked after S_q in the sorted list. According to Algorithm 2, there would be less than d_i feasible channels for S_i after the allocation for S_q. Therefore, S_i loses.

Thus, p_i is the critical value of S_i.

We have proved that the allocation algorithm of SPA is monotone, and the payment of each winner is its critical value. Therefore, SPA is truthful. ∎

Lemma 2. *SPA is individually rational.*

Proof: Assume that each buyer S_i bids truthfully, i.e., $b_i = v_i$. For each winning buyer S_i, Algorithm 2 returns $p_i = d_i \frac{\tilde{b}_q}{\tau_i}$. According to Equation (5), $u_i = v_i - p_i$. Because S_i is ranked before S_q, we have $\tilde{b}_i \geq \tilde{b}_q$. With $\tilde{b}_i = \frac{b_i}{d_i} \cdot \tau_i$ and $\tilde{b}_q = \frac{p_i}{d_i} \cdot \tau_i$, we obtain $b_i \geq p_i$. Therefore $u_i \geq 0$. For all losers, $u_i = 0$.

Thus, $u_i \geq 0$. SPA is individually rational. ■

Lemma 3. *SPA is computationally efficient.*

Proof: We now analyze the running time of SPA. First, Algorithm 1 takes $O(n \log n)$ time to sort the buyers, where n is the number of buyers. To allocate d_i channels to a buyer S_i, Algorithm 1 needs to examine at most m channels to find the feasible channels. This process takes $O(mn)$ time for n buyers. Therefore, the overall complexity of Algorithm 1 is $O(n \log n + mn)$. Second, Algorithm 2 uses the sorted bids from Algorithm 1 and hence its complexity only comes from the processes of initialization and checking feasibility for S_q, which is $O(mn)$ for each buyer. Therefore, the overall complexity of Algorithm 2 is $O(mn^2)$. In total, the overall complexity is $O(mn^2)$. ■

V. Performance Evaluation

In this section, we evaluate the performance of SPA by comparing it with an existing auction.

A. Environment Setup

As we surveyed in Section II, there is no existing auction under the physical interference model. Most of the known prior works [10, 14, 15, 18] form groups to achieve spatial reusability, according to the given conflict graphs. Thus they can be easily modified to adopt the physical interference model by using the SINR values to form groups, as Zhang *et.al.* did in TSA [20]. In this setting, we choose to compare SPA with SMALL [15], which is most related to our auction. Since the stage of group formation in SMALL is bid-independent, we implemented an effective heuristic algorithm for link scheduling in [16] to form secondary users into groups. We name the modified SMALL as **SMALL-SINR**.

To evaluate the performance of both SPA and SMALL-SINR, we uniformly distributed transmitters and receivers in a $1000m \times 1000m$ square region. The length of each link is randomly chosen between $100m$ and $200m$. The SINR threshold was set to 10, the environment noise $N_0 = 10^{-9}$, the path loss exponent $\alpha = 2$, and transmit power $P_0 = P_1 = \cdots = P_n = 0.2W$. We assume that the bids from all buyers are distributed uniformly at random over $(0, 100]$, and each buyer requests at most 3 channels. All results are averaged over 100 times for each parameter configuration.

B. Performance Metrics

We evaluate the performance of SPA using the following metrics.

- *Revenue*: The total payment from all the winning buyers.
- *Channel Utilization*: Average number of buyers allocated to each channel.
- *Buyer Satisfaction Ratio*: The percentage of buyers who win at least one channel.

C. Evaluation Results and Analysis

Figure 3 shows revenues, channel utilizations and buyer satisfaction ratios of both SPA and SMALL-SINR. The number of buyers is 100 and the number of auctioned channels varies from 5 to 85 with an increment of 5.

Figure 3(a) gives the trends of channel utilization for both SPA and SMALL-SINR. In SMALL-SINR, the average number of buyers in each channel stays at a steady level around 1.2. In SPA, the channel utilization is around 3 initially, and gradually falls to a level about 2.2. With more channels released, the competition among buyers in SPA becomes less intense, while SMALL always sacrifices one buyer in each group to maintain truthfulness.

In Figure 3(b), the satisfaction ratio of SMALL-SINR grows at first, then stays at a steady level. Because in SMALL-SINR, groups are formed before the allocation. When the number of channels increases to a certain value, the number of winning groups remains the same. In SPA, the satisfaction ratio increases with the number of channels, and is above 98% when there are enough channels for almost all buyers.

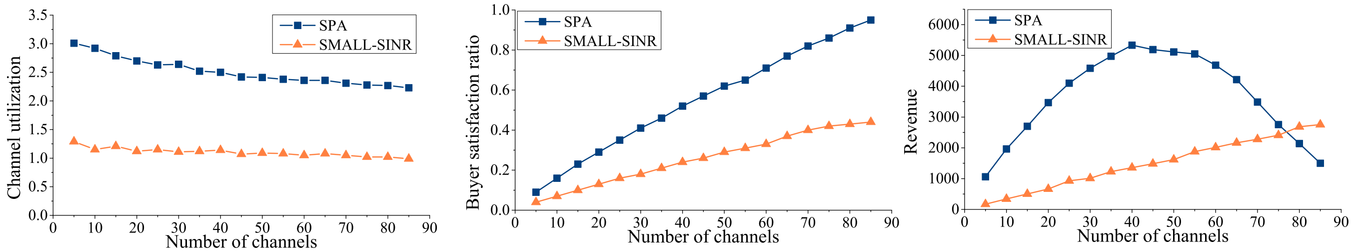
Form Figure 3(c) it can be observed that the revenue of SMALL-SINR grows with more channels involved, but the revenue converges after the saturation of the market. On the other hand, the revenue of SPA increases rapidly at the beginning and then falls down when the number of auctioned channels is above 40. The fundamental reason is that, with more channels, the competition among buyers is no longer intense, which leads to zero payments for some winners.

Figure 4 shows the impact of the number of SUs on revenues, channel utilizations and buyer satisfaction ratios for both SPA and SMALL-SINR. The number of auctioned channels is 50 and the number of SUs varies from 20 to 500.

Figure 4(a) illustrates the channel utilization when more buyers join the auctions. Based on the interference relationship, 2-3 buyers can share the same channel but SMALL-SINR always sacrifices one buyer in each group to maintain truthfulness. On the other hand, the group size is guaranteed to be no less than 1 due to the winner section rule. Therefore, channel utilization stays steady around 1.2 in SMALL-SINR. Whereas, in SPA the average number of buyers in each channel increases rapidly at first, and then remains at a level around 2.5 due to over-saturation of the market with more SUs.

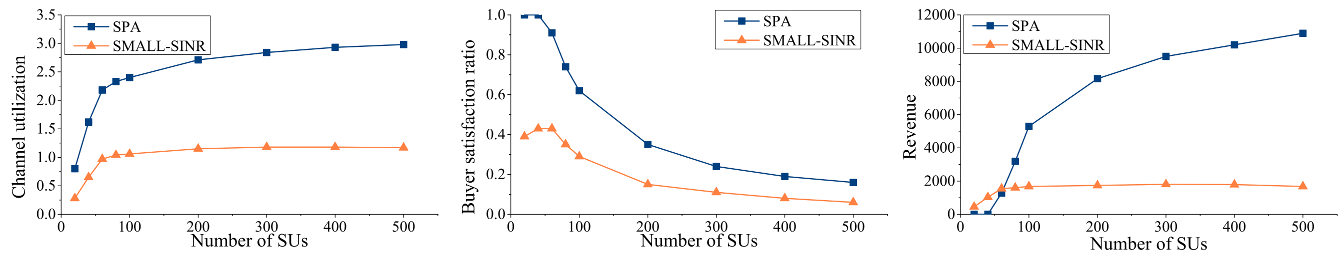
From Figure 4(b), it can be observed that, initially the satisfaction ratio is nearly 100% in SPA. Because most buyers are winners and the market is almost saturated. When more buyers join the auction, SPA does not provide sufficient channels to satisfy all the buyers. As a result, the satisfaction ratio drops. On the contrary, SMALL-SINR can hardly achieve high satisfaction ratio due to its sacrifice rule.

In Figure 4(c), the competition between buyers in SPA becomes more intense with more buyers involved. Consequently,



(a) Channel utilization (b) Buyer satisfaction ratio (c) Revenue

Figure 3. Comparing SPA and SMALL-SINR by auctioning 5-85 channels to 100 bidders.



(a) Channel utilization (b) Buyer satisfaction ratio (c) Revenue

Figure 4. Comparing SPA and SMALL-SINR by auctioning 50 channels to 20-500 buyers.

winners' critical values are higher and the revenue increases. Similarly, when the competition between groups in SMALL-SINR grows, the seller receives more revenue. However, the sacrifice rule in SMALL-SINR inhibits significant revenue growth, compared with that in SPA.

VI. Conclusion

In this paper, we proposed SPA, a spectrum auction that allows the primary and secondary users to share channels simultaneously under the physical interference model. We analyzed SPA and proved that it satisfies truthfulness, individual rationality, and computational efficiency. Further performance evaluation indicates SPA achieves better channel utilization and buyer satisfaction ratio compared with SMALL [15] adapted for the physical interference model.

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