Power Control for Cognitive Radio Systems with Unslotted Primary Users Under Sensing Uncertainty

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***Abstract*— This paper studies the optimal power control policy and frame duration that maximize the throughput of secondary users operating under transmit power, interference power, and collision constraints in the presence of unslotted primary users. It is assumed that primary user activity follows an ON-OFF alternating renewal process. Secondary users first sense the channel albeit with errors in the form of miss detections and false alarms, and then start the data transmission only if no primary user activity is detected. Under these assumptions, we determine the optimal power control policy subject to peak transmit power and average interference power constraints and propose a low-complexity algorithm for the joint optimization of the power level and frame duration under collision constraints. We further analyze some important properties of the collision duration ratio, which is defined as the ratio of average collision duration to transmission duration, and investigate the impact of the probabilities of detection and false alarm on the throughput, optimal transmission power, and the collisions with primary user transmissions.**

***Index Terms*—Cognitive radio, collision constraint, interference power constraint, optimal frame duration, optimal power control, probability of detection, probability of false alarm, renewal process, unslotted transmission.**

I. Introduction

Cognitive radio is a promising innovative technology, leading to more efficient spectrum management and utilization. In cognitive radio systems, unlicensed users (i.e., cognitive or secondary users) are allowed to either continuously share the spectrum licensed to legacy users (i.e., primary users) without causing any significant interference, or periodically monitor the primary user activity via spectrum sensing and then perform transmissions according to sensing decisions.

A. Motivation

Many existing works have focused on deriving the optimal power control policies and analyzing sensing-throughput trade-off in cognitive radio systems. In particular, the authors in [1] analyzed the optimal sensing duration and power control to maximize the ergodic capacity of cognitive radio systems operating in multiple narrowband channels under two different transmission schemes, namely sensing-based spectrum sharing and opportunistic spectrum access. The authors in [2] characterized the effective capacity of secondary users and the corresponding optimal power control policy in the presence of sensing errors. In these works, it is assumed that primary users transmit in a time-slotted fashion, i.e., the activity of the primary users (e.g., active or inactive) remains the same during the entire frame duration.

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In practice, primary and secondary user transmissions may not necessarily be synchronized. For instance, the primary user traffic can be bursty and may change its status during the transmission phase of the secondary users. In such cases, the assumption of time-slotted primary user transmission adopted in most studies (as also seen in the above-mentioned works) does no longer hold. In unslotted scenarios, it is assumed that ON-OFF periods of the primary user transmissions are random variables, following certain specific distributions. Exponential distribution is a commonly used model (see e.g., [3] – [5]). In particular, the authors in [3] determined the optimal frame duration that maximizes the throughput of the secondary users with perfect sensing decisions under collision constraints, assuming that the primary user activity changes only once within each frame. By adopting the same assumptions for the primary user activity as in the previous work, the authors in [4] mainly focused on the throughput of secondary users operating in the presence of multiple primary users with imperfect channel sensing results. In the same setting, the work in [5] mainly analyzed the optimal frame duration that maximizes the secondary user throughput. In [6], the exact secondary user throughput was determined and joint optimization of the sensing duration and frame period in the presence of sensing errors was performed by assuming that the primary user changes its status multiple times.

B. Main Contributions

Motivated mainly by the fact that the optimal power control policies have not been derived in the presence of unslotted primary users, we have the following key contributions in this paper:

- We derive, in closed-form, the optimal power control policy which maximizes the throughput of the secondary users operating with unslotted primary users subject to peak transmit power, average interference power and collision constraints in the presence of sensing errors. We do not impose any limitations on the number of transitions of the primary user activity unlike the studies in [3], [4], [5] where the primary user activity changes only once. We assume that the primary user can change its status between ON and OFF states multiple times.
- We propose a low-complexity algorithm for jointly finding the optimal power control policy and the frame duration.

- We analyze some important properties of the collision duration ratio and the relations among sensing performance, secondary user throughput, optimal frame duration and the resulting collisions with the primary user.

II. System Model

In this paper, we consider a cognitive radio system consisting of a pair of primary transmitter and receiver, and a pair of secondary transmitter and receiver. Secondary users opportunistically access the channel licensed to the primary users. In the following subsections, we describe the primary user activity model, opportunistic spectrum access policy of the secondary users, and the formulation of the collision constraint imposed for the protection of the primary users.

A. Primary User Activity Model

Differing from the majority of the studies (which assume that the primary users adopt a time-slotted transmission scheme), we consider a continuous, i.e., unslotted transmission structure as shown in Figure 1.

We assume that the primary user activity follows a semi-Markov process with ON and OFF states, which is shown to be a good model for primary user traffic based on measurements and simulations [7], [8]. In this model, the ON state indicates that the primary user is transmitting while the OFF state represents that the channel is not occupied by the primary user. Such a process is also known as an alternating renewal process. The durations of ON and OFF periods are independent of each other and are exponentially distributed with means λ_0 and λ_1, respectively, and therefore have probability density functions

$$f_{\text{ON}}(t) = \frac{1}{\lambda_0} \mathrm{e}^{-\frac{t}{\lambda_0}}, \text{ and } f_{\text{OFF}}(t) = \frac{1}{\lambda_1} \mathrm{e}^{-\frac{t}{\lambda_1}}. \quad (1)$$

Hence, the prior probabilities of channel being vacant or occupied by the primary user can be expressed, respectively, as

$$P(\mathcal{H}_0) = \frac{\lambda_0}{\lambda_0 + \lambda_1}, \qquad P(\mathcal{H}_1) = \frac{\lambda_1}{\lambda_0 + \lambda_1}. \quad (2)$$

B. Opportunistic Spectrum Access by the Secondary Users

Secondary users employ frames of duration T_{f}. In the initial duration of τ seconds, secondary users perform channel sensing and monitor the primary user activity. Subsequently, data transmission starts in the remaining frame duration of $T_{\text{f}} - \tau$ seconds only if the primary user activity is not detected, the event of which is denoted by $\hat{\mathcal{H}}_0$. Spectrum sensing is modeled as a simple binary hypothesis testing problem with two hypotheses $\mathcal{H}_0$ and $\mathcal{H}_1$ corresponding to the absence and presence of the primary user signal, respectively. Many spectrum sensing methods have been proposed in the literature [9], [10] and the corresponding sensing performance is characterized by two parameters, namely the probabilities of detection and false alarm, which are defined as

$$P_{\text{d}} = \Pr\{\hat{\mathcal{H}}_1 | \mathcal{H}_1\}, \quad P_{\text{f}} = \Pr\{\hat{\mathcal{H}}_1 | \mathcal{H}_0\}, \quad (3)$$

where $\hat{\mathcal{H}}_1$ denotes the event that the primary user activity is detected. Any sensing method can be employed in the rest of the analysis since the results depend on the sensing performance only through the probabilities of detection and false alarm, and the sensing duration.

C. Collision Constraints

We first describe the secondary users' collisions with the primary users, which can lead to considerable performance degradation in the primary user transmissions. Subsequently, we impose a constraint on the ratio of the average collision duration to the transmission duration in order to protect the primary users. Depending on the true nature of the primary user activity at the beginning of the frame, collisions between the primary and secondary users can occur in the following two cases:

- *Case* 1: The channel is not occupied by the primary user and is correctly detected as idle at the beginning of the frame. Even if the primary user is not actually transmitting initially, it is possible for the primary user to start data transmission at any time during the current frame, which results in a collision event. By conditioning on the correct detection of the initial absence of the primary user, the ratio of the average collision duration to data transmission duration, which is called the collision duration ratio, can be expressed as

$$P_{c,0} = \frac{\mathbb{E}\{T_{c|\mathcal{H}_0, \hat{\mathcal{H}}_0}\}}{T_{\text{f}} - \tau}, \quad (4)$$

where $\mathbb{E}\{\cdot\}$ denotes expectation operation and $T_{c|\mathcal{H}_0, \hat{\mathcal{H}}_0}$ is a random variable representing the collision duration between the secondary and primary users given that the primary user is inactive initially at the beginning of the frame (event $\mathcal{H}_0$) and the sensing decision is idle (event $\hat{\mathcal{H}}_0$). Assuming that the primary user is in the OFF state at first and taking into account the possible multiple transitions between ON and OFF states, $\mathbb{E}\{T_{c|\mathcal{H}_0, \hat{\mathcal{H}}_0}\}$ can be found by following a similar analysis as in [11, Theorem 2]. Hence, $P_{c,0}$ is given by

$$P_{c,0} = P(\mathcal{H}_1) - \frac{\lambda_0 P(\mathcal{H}_1)^2}{T_{\text{f}} - \tau} \left(1 - \mathrm{e}^{-\frac{T_{\text{f}} - \tau}{\lambda_0 P(\mathcal{H}_1)}}\right). \quad (5)$$

- *Case* 2: The primary user is actually present in the channel at the beginning of the frame, however the secondary user miss-detects the primary user activity, resulting in a collision right away due to sensing error. Multiple collisions can also occur if the primary user turns OFF and then back ON in a single frame once or multiple times. Similar to the first case, by conditioning on the miss detection event, the collision duration ratio can be found as

$$P_{c,1} = \frac{\mathbb{E}\{T_{c|\mathcal{H}_1, \hat{\mathcal{H}}_0}\}}{T_{\text{f}} - \tau} \quad (6)$$

$$= P(\mathcal{H}_1) + \frac{\lambda_1 P(\mathcal{H}_0)^2}{T_{\text{f}} - \tau} \left(1 - \mathrm{e}^{-\frac{T_{\text{f}} - \tau}{\lambda_0 P(\mathcal{H}_1)}}\right) \quad (7)$$

where $T_{c|\mathcal{H}_1, \hat{\mathcal{H}}_0}$ is a random variable describing the collision duration between the secondary and primary users given that the primary user is active at the beginning of the frame but sensing decision is incorrectly an idle channel.

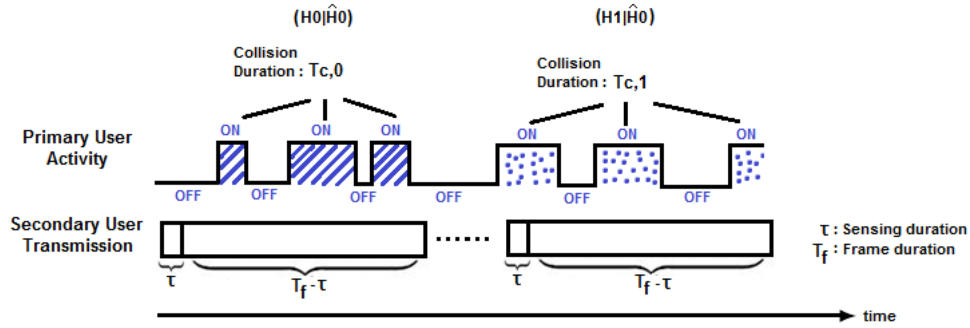


Fig. 1: Frame structure of the primary and secondary users.

Based on the above two cases, the collision duration ratio averaged over the true nature of the primary user activity given the idle sensing decision $\hat{\mathcal{H}}_0$ can be expressed as

$$P_c = P(\mathcal{H}_0|\hat{\mathcal{H}}_0)P_{c,0} + P(\mathcal{H}_1|\hat{\mathcal{H}}_0)P_{c,1} \tag{8}$$

where $P(\mathcal{H}_0|\hat{\mathcal{H}}_0)$ and $P(\mathcal{H}_1|\hat{\mathcal{H}}_0)$ denote the conditional probabilities of the primary user being active or inactive given the idle sensing decision, respectively, which can be written in terms of P_d and P_f as

$$P(\mathcal{H}_0|\hat{\mathcal{H}}_0) = \frac{P(\mathcal{H}_0)(1-P_\mathrm{f})}{P(\mathcal{H}_0)(1-P_\mathrm{f}) + P(\mathcal{H}_1)(1-P_\mathrm{d})}, \tag{9}$$

$$P(\mathcal{H}_1|\hat{\mathcal{H}}_0) = \frac{P(\mathcal{H}_1)(1-P_\mathrm{d})}{P(\mathcal{H}_0)(1-P_\mathrm{f}) + P(\mathcal{H}_1)(1-P_\mathrm{d})}. \tag{10}$$

In the following, we provide two key properties of P_c.

Proposition 1. *The average collision duration ratio P_c under idle sensing decision has the following properties:*

- *It is an increasing function of the frame duration T_f for $P_\mathrm{f} < P_\mathrm{d}$ and a decreasing function for $P_\mathrm{f} > P_\mathrm{d}$.*
- *It takes values between $P(\mathcal{H}_1|\hat{\mathcal{H}}_0)$ and $P(\mathcal{H}_1)$.*

Proof: The first derivative of P_c with respect to frame duration T_f is

$$P_c' = \left(P(\mathcal{H}_0|\hat{\mathcal{H}}_0)\lambda_0 P(\mathcal{H}_1)^2 - P(\mathcal{H}_1|\hat{\mathcal{H}}_0)\lambda_1 P(\mathcal{H}_0)^2\right) \times \left(\frac{1-\mathrm{e}^{-\frac{T_\mathrm{f}-\tau}{\lambda_0 P(\mathcal{H}_1)}}}{(T_\mathrm{f}-\tau)^2} - \frac{1}{\lambda_0 P(\mathcal{H}_1)(T_\mathrm{f}-\tau)}\mathrm{e}^{-\frac{T_\mathrm{f}-\tau}{\lambda_0 P(\mathcal{H}_1)}}\right). \tag{11}$$

The expression inside the first parenthesis can easily be seen to be greater than zero if $P_\mathrm{f} < P_\mathrm{d}$ and less than zero if $P_f > P_\mathrm{d}$ by using the formulations in (2), (9) and (10). In order to show that the expression inside the second parenthesis is always nonnegative, we compare it with zero as in the following:

$$\frac{1-\mathrm{e}^{-\frac{T_\mathrm{f}-\tau}{\lambda_0 P(\mathcal{H}_1)}}}{(T_\mathrm{f}-\tau)^2} - \frac{1}{\lambda_0 P(\mathcal{H}_1)(T_\mathrm{f}-\tau)}\mathrm{e}^{-\frac{T_\mathrm{f}-\tau}{\lambda_0 P(\mathcal{H}_1)}} > 0. \tag{12}$$

Above inequality can be rewritten as

$$\left(1 + \frac{T_\mathrm{f}-\tau}{\lambda_0 P(\mathcal{H}_1)}\right)\mathrm{e}^{-\frac{T_\mathrm{f}-\tau}{\lambda_0 P(\mathcal{H}_1)}} < 1. \tag{13}$$

Left-hand side of (13) is a decreasing function since its first derivative with respect to frame duration T_f is $-\frac{T_\mathrm{f}-\tau}{(\lambda_0 P(\mathcal{H}_1))^2}\mathrm{e}^{-\frac{T_\mathrm{f}-\tau}{\lambda_0 P(\mathcal{H}_1)}} \leq 0$. Since it is a decreasing function and it takes values between $(0,1)$ for $T_\mathrm{f} > \tau$, the inequality in (13) and hence the inequality in (12) hold. With this, we have shown that the expression inside the second parenthesis in (11) is nonnegative, and therefore the first derivative of P_c is greater than zero if $P_\mathrm{f} < P_\mathrm{d}$ and less than zero if $P_\mathrm{f} > P_\mathrm{d}$, proving the property that P_c is increasing with T_f if $P_\mathrm{f} < P_\mathrm{d}$ and decreasing with T_f if $P_\mathrm{f} > P_\mathrm{d}$.

Also, it can be easily verified that P_c takes values between $P(\mathcal{H}_1|\hat{\mathcal{H}}_0)$ and $P(\mathcal{H}_1)$ by examining the limit of P_c as T_f approaches τ and ∞, respectively. □

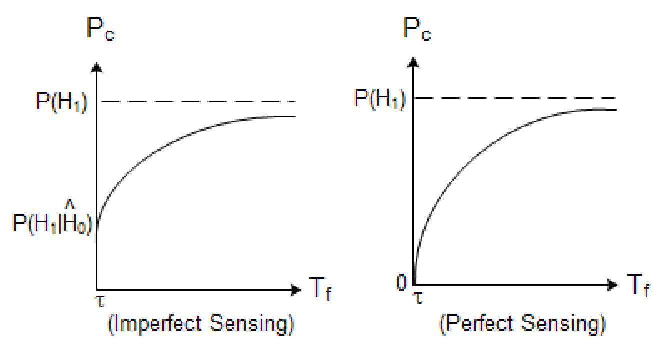


Fig. 2: Average collision duration vs. frame duration, T_f for imperfect sensing and perfect sensing cases.

In Fig. 2, we illustrate P_c as a function of the frame duration T_f when $P_\mathrm{f} < P_\mathrm{d}$, i.e., correct detection probability is greater than the false alarm probability. Note that this is generally the case in practice in which the probability of detection is expected to be greater than 0.5 and the probability of false alarm be less than 0.5 for reliable sensing performance. Both imperfect sensing and perfect sensing are considered. For the case of imperfect sensing, P_c takes values between $P(\mathcal{H}_1|\hat{\mathcal{H}}_0)$ and $P(\mathcal{H}_1)$. For perfect sensing, P_c is first 0 since $P(\mathcal{H}_1|\hat{\mathcal{H}}_0) = 0$, which corresponds to no collision event initially, as expected, and then P_c starts increasing with increasing T_f as it becomes more likely that the primary user initiates a transmission and secondary users collide with the primary users.

III. Optimal Power Control and Frame Duration

In this section, we find the optimal power control policy and the frame duration that maximize the throughput of the secondary users operating subject to interference power and collision constraints in the presence of sensing uncertainty and unslotted primary users. The optimization problem can be formulated as

$$\max_{T_\mathrm{f},P_0} R=\frac{T_\mathrm{f}-\tau}{T_\mathrm{f}}\mathbb{E}\Big\{P(\mathcal{H}_0)(1-P_\mathrm{f})\Big[\log_2\Big(1+\frac{P_0|h|^2}{N_0}\Big)(1-P_{c,0}) + \log_2\Big(1+\frac{P_0|h|^2}{N_0+\sigma_s^2}\Big)P_{c,0}\Big] + P(\mathcal{H}_1)(1-P_\mathrm{d}) \times\Big[\log_2\Big(1+\frac{P_0|h|^2}{N_0}\Big)(1-P_{c,1}) + \log_2\Big(1+\frac{P_0|h|^2}{N_0+\sigma_s^2}\Big)P_{c,1}\Big]\Big\} \tag{14}$$

subject to

$$P_c \leq P_{c,max} \tag{15}$$

$$\frac{T_\mathrm{f}-\tau}{T_\mathrm{f}}\mathbb{E}\{P_0(P(\mathcal{H}_0,\hat{\mathcal{H}}_0)P_{c,0}+P(\mathcal{H}_1,\hat{\mathcal{H}}_0)P_{c,1})|g|^2\} \leq Q_\mathrm{avg} \tag{16}$$

$$P_\mathrm{pk} \geq P_0 \geq 0 \tag{17}$$

$$T_\mathrm{f} \geq \tau \tag{18}$$

where h and g denote the channel fading coefficients of the transmission link between the secondary transmitter and the secondary receiver and the interference link between the secondary transmitter and the primary receiver, respectively. It is assumed that the secondary transmitter has perfect channel side information of h and g. More specifically, the secondary receiver's channel estimate h is sent to the secondary transmitter via an error-free feedback link. Also, the knowledge of the interference link, g, can be obtained through direct feedback from the primary receiver [12], indirect feedback from a third party such as a band manager [13] or by periodically sensing the pilot symbols sent by the primary receiver under the assumption of channel reciprocity [14]. In addition, N_0 and σ_s^2 in the above formulations represent the variances of the additive noise and the primary user's received faded signal.

In (15), $P_{c,max}$ denotes the maximum tolerable collision duration ratio, which needs to be greater than $P(\mathcal{H}_1|\hat{\mathcal{H}}_0)$ based on Proposition 1 because, otherwise, the constraint cannot be satisfied. Since P_c is an increasing function of T_f when $P_\mathrm{f} < P_\mathrm{d}$, the collision constraint in (15) provides an upper bound on the frame duration T_f as follows:

$$T_\mathrm{f} \leq P_c^{-1}(P_{c,max}). \tag{19}$$

Above, $P_c^{-1}(.)$ is the inverse function of P_c. In addition to the collision constraint in (15), in order to satisfy the long-term QoS requirements of the primary users, we further impose an interference power constraint in (16), where Q_{avg} represents the average received interference power limit at the primary receiver.

The throughput of the secondary user denoted by R in (14) is concave with respect to transmission power P_0 but is not a concave function of the frame duration T_f since the second derivative of R with respect T_f is less than, greater than or equal to zero depending on the sensing parameters and prior probabilities of primary user being active and idle. However, the optimal frame duration which maximizes the throughput can easily be obtained using a one-dimensional exhaustive search within the interval $(\tau, P_c^{-1}(P_{c,max})]$. For a given frame duration, we derive the optimal power control policy in the following result.

Theorem 1. *The optimal power control that maximizes the throughput of the secondary users operating subject to the average interference power constraint in (16) and peak power constraint (17) in the presence of sensing errors and unslotted primary users is given by*

$$P_0^* = \min\left\{\left[\frac{A_0+\sqrt{\Delta_0}}{2}\right]^+, P_\mathrm{pk}\right\} \tag{20}$$

where

$$A_0 = \frac{P(\hat{\mathcal{H}}_0)\log_2(e)}{\mu\alpha_0|g|^2} - \frac{2N_0+\sigma_s^2}{|h|^2} \tag{21}$$

$$\Delta_0 = A_0^2 - \frac{4}{|h|^2}\left(\frac{N_0(N_0+\sigma_s^2)}{|h|^2} - \frac{(P(\hat{\mathcal{H}}_0)(N_0+\sigma_s^2)-\alpha_0\sigma_s^2)\log_2(e)}{\mu\alpha_0|g|^2}\right) \tag{22}$$

$$\alpha_0 = P(\mathcal{H}_0,\hat{\mathcal{H}}_0)P_{c,0} + P(\mathcal{H}_1,\hat{\mathcal{H}}_0)P_{c,1} \tag{23}$$

where $(x)^+ = \max\{0,x\}$ and μ is the Lagrange multiplier, which can be determined by satisfying the average interference power constraint in (16) with equality.

Proof: The objective function in (14) is strictly concave since it is composed of positive weighted sum of logarithms which are strictly concave. Hence, the optimal power can be obtained by using the Lagrangian optimization approach as follows:

$$L(P_0,\mu) = R + \mu\Big(Q_{avg} - \frac{T_\mathrm{f}-\tau}{T_\mathrm{f}}\mathbb{E}\{P_0(P(\mathcal{H}_0,\hat{\mathcal{H}}_0)P_{c,0} + P(\mathcal{H}_1,\hat{\mathcal{H}}_0)P_{c,1})|g|^2\}\Big) \tag{24}$$

where μ is the nonnegative Lagrange multiplier. The Lagrange dual problem is defined as

$$\min_{\mu\geq 0}\max_{0\leq P_0\leq P_\mathrm{pk}} L(P_0,\mu). \tag{25}$$

For a fixed μ and each fading state, we express the sub-problem using the Lagrange dual decomposition method [15]. According to the Karush-Kuhn-Tucker (KKT) conditions, the optimal power control P_0^* must satisfy the set of equations and inequalities in (26) – (29) shown at top of the next page. It is observed that if $\mathbb{E}\{|g|^2\} \leq \frac{T_\mathrm{f}}{T_\mathrm{f}-\tau}\frac{Q_\mathrm{avg}}{\alpha_0 P_\mathrm{pk}}$, then the average interference power constraint in (16) is loose. Hence, $\mu = 0$ and $P_0^* = P_\mathrm{pk}$. If $\mathbb{E}\{|g|^2\} > \frac{T_\mathrm{f}}{T_\mathrm{f}-\tau}\frac{Q_\mathrm{avg}}{\alpha_0 P_\mathrm{pk}}$, then $\mu > 0$. Therefore, by solving (26), incorporating the nonnegativity of the transmit power, and combining with the peak power constraint yield the desired result in (20). □

The value of μ can be obtained numerically via the projected subgradient method. In this method, μ is updated iteratively

$$\Big(\frac{P(\hat{\mathcal{H}}_0)-\alpha_0}{\log(2)}\Big)|h|^2(N_0+\sigma_s^2+P_0|h|^2)+\frac{\alpha_0}{\log(2)}|h|^2(N_0+P_0|h|^2)-(\mu\alpha_0|g|^2)(N_0+P_0|h|^2)(N_0+P_0|h|^2)=0 \quad (26)$$

$$\mu\Big(\frac{T_{\rm f}-\tau}{T_{\rm f}}\mathbb{E}\{P_0(P(\mathcal{H}_0,\hat{\mathcal{H}}_0)P_{c,0}+P(\mathcal{H}_1,\hat{\mathcal{H}}_0)P_{c,1})|g|^2\}-Q_{\rm avg}\Big)=0, \quad (27)$$

$$\mu\geq 0, \quad (28)$$

$$\frac{T_{\rm f}-\tau}{T_{\rm f}}\mathbb{E}\{P_0(P(\mathcal{H}_0,\hat{\mathcal{H}}_0)P_{c,0}+P(\mathcal{H}_1,\hat{\mathcal{H}}_0)P_{c,1})|g|^2\}-Q_{\rm avg}\leq 0. \quad (29)$$

$$\mu^{(n+1)}=\Big(\mu^{(n)}-t\big(Q_{\rm avg}-\frac{T_{\rm f}-\tau}{T_{\rm f}}\mathbb{E}\{P_0(P(\mathcal{H}_0,\hat{\mathcal{H}}_0)P_{c,0}+P(\mathcal{H}_1,\hat{\mathcal{H}}_0)P_{c,1})|g|^2\}\big)\Big)^+ \quad (30)$$

TABLE I

Algorithm 1 The optimal power control and frame duration algorithm that maximize the throughput of the secondary user under the peak transmit power, average interference power, and collision constraints

Initialize $\epsilon>0$, $t>0$, $\mu^{(0)}=\mu_{\rm init}$, $P_{c,max}=P_{c,max,{\rm init}}$
if $P_{c,max}<P(\mathcal{H}_1|\hat{\mathcal{H}}_0)$ **then**
  $T_{\rm f,opt}=0$, $P_0^*=0$
else
  For $T_{\rm f}=(\tau:P_c^{-1}(P_{c,max})]$
  if the average interference power constraint in (16) is satisfied when $P_0^*=P_{\rm pk}$ **then**
    $P_0^*=P_{\rm pk}$
  else
    repeat
      $n\leftarrow 0$
      $P_0^*=\max\left(P_{\rm pk},P_0^*\right)$ using (20)
      Update μ using the projected subgradient method shown in (30)
    until $\left|\mu^{(n)}(Q_{\rm avg}-\frac{T_{\rm f}-\tau}{T_{\rm f}}\mathbb{E}\{P_0(P(\mathcal{H}_0,\hat{\mathcal{H}}_0)P_{c,0}+P(\mathcal{H}_1,\hat{\mathcal{H}}_0)P_{c,1})|g|^2\})\right|\leq\epsilon$
  end if
  Calculate throughput R using (14),
  $T_{\rm f,opt}=\arg\max R$
  $P_{0,opt}^*=[P_0^*]_{T_{\rm f}=T_{\rm f,opt}}$
end if

in the direction of a negative subgradient of the Lagrangian function $L(P_0,\mu)$ in (24) until convergence as shown in (30), where n is the iteration index and t is the step size. For a constant t, μ is shown to converge to the optimal value within a small range [16].

In Table I, we propose a low-complexity algorithm for jointly finding the optimal power control policy and the frame duration, which maximize throughput of the secondary user in the presence of unslotted primary users and imperfect sensing decisions.

IV. Numerical Results

In this section, we present and discuss several numerical results for the optimal power control and frame duration, which maximize the throughput of the secondary users, and analyze the resulting collisions with the unslotted primary users. Unless mentioned explicitly, it is assumed that the noise variance is $N_0=0.01$ and the variance of primary user's received signal is $\sigma_s^2=0.1$. Also, λ_0 and λ_1 are set to 650 ms and 352 ms, respectively so that $P(\mathcal{H}_0)\approx 0.65$, which corresponds to voice over Internet protocol (VOIP) traffic. The sensing duration τ is 1 ms, the step size t is chosen as 0.1, and tolerance ϵ is set to 0.00001. In addition, the peak transmit power limit is $P_{\rm pk}=10$ dB and we consider Rayleigh fading.

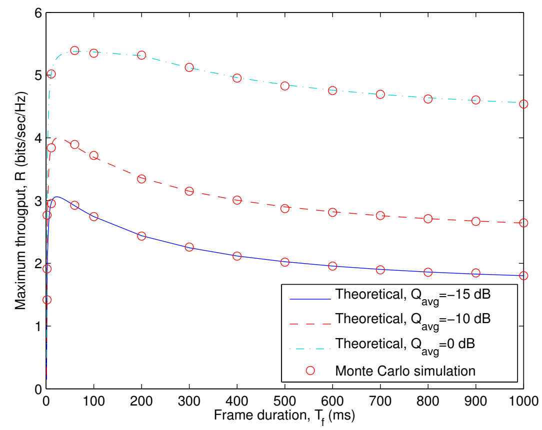
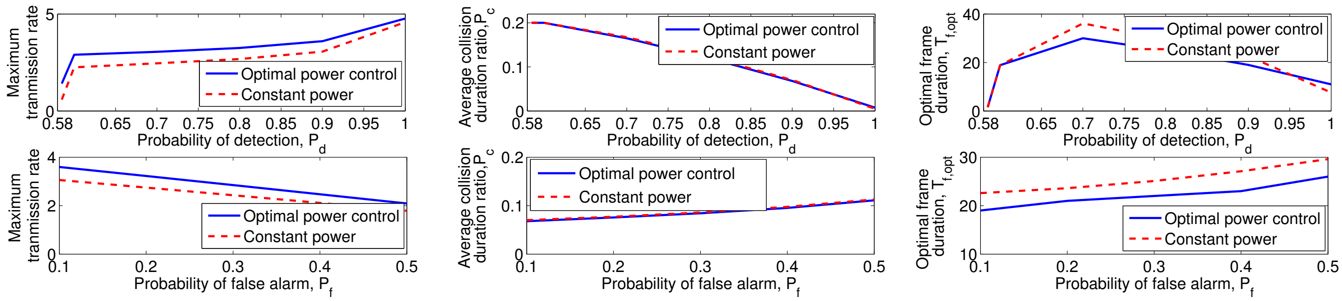


Fig. 3: Throughput of secondary users, R vs. frame duration, $T_{\rm f}$.

In Fig. 3, we plot the throughput of the secondary users, R, as a function of the frame duration $T_{\rm f}$ for different average power constraints, namely $Q_{\rm avg}=-15$ dB, $Q_{\rm avg}=-10$ dB and $Q_{\rm avg}=0$ dB. We consider imperfect sensing with $P_{\rm d}=0.9$ and $P_{\rm f}=0.1$. Transmission power level P_0 is chosen according to $\min\left(P_{\rm pk},\frac{T_{\rm f}}{T_{\rm f}-\tau}\frac{Q_{\rm avg}}{(P(\mathcal{H}_0,\hat{\mathcal{H}}_0)P_{c,0}+P(\mathcal{H}_1,\hat{\mathcal{H}}_0)P_{c,1})}\right)$. In this setting, throughput formulation in (14) is verified through Monte Carlo simulations with 100000 runs. It is seen that R initially increases with increasing transmission duration. After reaching a peak value, R starts decreasing as the secondary user starts colliding with primary user transmissions more frequently, degrading the performance. It is also observed that as the interference power constraint gets looser, i.e., as $Q_{\rm avg}$ changes from -15 to 0 dB, higher throughput is achieved since secondary user transmits at higher power levels. As illustrated in the figure, R is not a concave function of $T_{\rm f}$. However, R curves are seen to exhibit a quasiconcave property and there exists an optimal frame duration that maximizes the throughput.

In Fig. 4, we display the maximum secondary user throughput



(a) (b) (c)

Fig. 4: (a) Maximum throughput of secondary users vs. P_{d} and probability of false alarm, P_{f} (b) Average collision duration ratio, P_c vs. P_{d} and P_{f} (c) Optimal frame duration, $T_{\mathrm{f,opt}}$ vs. P_{d} and P_{f}.

R, average collision duration ratio P_c, and the optimal frame duration $T_{\mathrm{f,opt}}$ as a function of the probability of detection P_{d} and the probability of false alarm P_{f}. We set the maximum collision limit as $P_{c,max} = 0.2$. Q_{avg} is chosen as -15 dB. We both consider the optimal power control policy and the constant power transmission. For the constant power case, transmission power P_0 is chosen as the same value as in the previous figure, and hence power is not adaptively varied with respective to the channel fading coefficients of the transmission link and interference link, denoted by h and g, respectively. On the other hand, optimal power control derived in (20) is a function of both h and g. As P_{d} increases while keeping P_{f} fixed to 0.1 and hence sensing performance improves, secondary user has a higher throughput and lower collision duration ratio. For P_{d} values less than 0.585, collision constraint is not satisfied for any value of the frame duration T_{f}, therefore the secondary user throughput is 0. When P_{d} takes values between 0.585 and 0.6, maximum throughput is achieved at the maximum collision limit, i.e, when $P_c = 0.2$. It is also observed that the optimal power control provides higher throughput and results in less collisions compared to constant power transmissions.

In the same figure, it is seen that as P_{f} increases while keeping P_{d} fixed to 0.9, sensing performance degrades and secondary users experience more false alarm events, which leads to more collisions with the primary users. Therefore, secondary user has a lower throughput and more frequent collisions with the primary user transmissions for both optimal power control and constant power cases. We also notice in the figure that the optimal power control outperforms constant power transmissions.

V. Conclusion

In this paper, we have obtained the optimal power control policy and frame duration for the secondary users operating with unslotted primary users subject to peak transmit power, average interference power and collision constraints in the presence of sensing errors. We have also provided a low-complexity optimal power control and frame duration algorithm. Numerical results reveal that optimal power control outperforms constant power scheme. We have also addressed how secondary user throughput, collisions with the primary user transmissions, and the optimal frame duration vary as a function of the probabilities of detection and false alarm.

References

[1] S. Stotas and A. Nallanathan, "Optimal sensing time and power allocation in multiband cognitive radio networks," *IEEE Trans. Wireless Commun.*, vol. 59, no. 1, pp. 226-235, Jan. 2011.

[2] S. Akin and M. C. Gursoy, "Effective capacity analysis of cognitive radio channels for quality of service provisioning," *IEEE Trans. Wireless Commun.*, vol. 9, no. 11, pp. 3354-3364, Nov. 2010.

[3] Y. Pei, A. T. Hoang, and Y. C. Liang, "Sensing-throughput tradeoff in cognitive radio networks: how frequently should spectrum sensing be carried out," in *Proc. of IEEE International Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC)*, pp. 1-5, Sept. 2007.

[4] W. Tang, M. Z. Shakir, M. A. Imran, R. Tafazolli, and M. S. Alouini, "Throughput analysis for cognitive radio networks with multiple primary users and imperfect spectrum sensing," *Communications, IET*, vol. 6, no. 17, pp. 2787-2795, 2012.

[5] S. Zarrini and T. J. Lim, "Throughput-sensing tradeoff of cognitive radio networks based on quickest sensing," in *Proc. of IEEE International Conference on Communications (ICC)*, pp. 1-5, June 2011.

[6] M. Guerrini, L. Rugini, and P. Banelli, "Sensing-throughput tradeoff for cognitive radios," in *Proc. of IEEE Workshop on Signal Processing Advances in Wireless Communications (SPAWC)*, pp. 115-119, June 2013.

[7] J. Riihijärvi, J. Nasreddine, and P. Mähönen, "Impact of primary user activity patterns on spatial spectrum reuse opportunities," in *Proc. of IEEE Wireless Conference (EW)*, pp. 962-968, April 2010.

[8] M. Wellens, J. Riihijärvi, and P. Mähönen, "Empirical time and frequency domain models for spectrum use," *Phys. Commun.*, vol. 2, no. 1/2, pp. 10-32, 2009.

[9] A. Ghasemi and E. S. Sousa, "Spectrum sensing in cognitive radio networks: requirements, challenges and design trade-offs," *IEEE Comm. Mag.*, vol. 46, no, 4, pp. 32-39, Apr. 2008.

[10] E. Axell, G. Leus, E. G. Larsson, and H. V. Poor, "Spectrum sensing for cognitive radio: State-of-the-art and recent advances," *IEEE Signal Process. Mag.*, vol. 29, no. 3, pp. 101-116, May 2012.

[11] C. Jiang, Y. Chen, K. J. R. Liu, and Y. Ren, "Renewal-theoretical dynamic spectrum access in cognitive radio network with unknown primary behavior," *IEEE J. Sel. Areas in Commun.*, vol. 31, no. 3, pp. 406-416, March 2013.

[12] M. -S. Alouini, and A. J. Goldsmith, "Capacity of Rayleigh fading channels under different adaptive transmission and diversity-combining techniques," *IEEE Trans. Veh. Tech.*, vol. 48, no. 4, pp. 1165-1181, July 1999.

[13] J. M. Peha, "Approaches to spectrum sharing," *IEEE Commun. Mag.*, vol. 43, no. 2, pp. 10-12, Feb. 2005.

[14] Q. Zhao, S. Geirhofer, L. Tong, and B. M. Sadler, "Opportunistic spectrum access via periodic channel sensing," *IEEE Trans. Signal Process.*, vol. 56, no. 2, pp. 785-796, Feb. 2008.

[15] D. Palomar and M. Chiang, "A tutorial on decomposition methods for network utility maximization," *IEEE J. on Sel. Areas in Commun.*, vol. 24, no. 8, pp. 1439 1451, Aug. 2006.

[16] S. Boyd, L. Xiao, and A. Mutapcic, "Subgradient methods," Lecture Notes of EE392o, Standford University, Autumn Quarter 2003-2004.