Radio-as-a-Service: Auction-based Model and Mechanisms

Jing Wang, Dejun Yang, Jian Tang and Mustafa Cenk Gursoy

***Abstract*—We envision that in the near future, just as Infrastructure-as-a-Service (IaaS), radios and radio resources in a wireless network can also be provisioned as a service to Mobile Virtual Network Operators (MVNOs), which we refer to as *Radio-as-a-Service (RaaS)*. In this paper, we present a novel auction-based model to enable fair pricing and fair resource allocation according to real-time needs of MVNOs for RaaS. Based on the proposed model, we study the auction mechanism design with the objective of maximizing social welfare. First, we present an Integer Linear Programming (ILP) based auction mechanism for obtaining optimal social welfare. To reduce time complexity, we present a polynomial-time greedy mechanism for the RaaS auction. Both methods have been formally shown to be truthful and individually rational. Extensive simulation results show that the proposed greedy auction mechanism can quickly produce close-to-optimal solutions.**

***Index Terms*–Cloud Computing, Radio-as-a-Service, Auction**

I. Introduction

Inspired by the success of application of Virtual Machines (VMs) in cloud computing, virtualization has been introduced to wireless networking recently [6], enabling support for multiple Mobile Virtual Network Operators (MVNOs) via isolated slices over a shared wireless substrate.

We envision that in the near future, just as Infrastructure-as-a-Service (IaaS), radios and radio resources in a wireless network can also be provisioned as a service to multiple MVNOs, which we refer to as *Radio-as-a-Service (RaaS)*. In an RaaS cloud, the cloud service provider owns radios, i.e., Base Stations (BSs) lease radio resources to MVNOs for profit. Similar to a tenant in an IaaS cloud, an MVNO pays the cloud service provider to use radio resources to serve its own users. Radio resources are usually shared among multiple MVNOs. For wide adoption of RaaS, on one hand, the cloud service provider needs to be able to collect a fair amount of payment from each MVNO for radio resources it leases; on the other hand, an MVNO needs to be able to obtain sufficient resources from the cloud service provider to well serve its users at a fair cost. In an IaaS cloud (such as Amazon EC2), resources are given to tenants in the format of VM and storage space, which has guaranteed capabilities/capacities for computing and storage respectively. However, in an RaaS, bandwidth (i.e., transmission capability) of a wireless link is time-varying. An MVNO, which rents a certain amount of radio resources beforehand, may not have sufficient bandwidth for its users in certain periods of time. Hence, supporting RaaS is very challenging but has not yet been well studied.

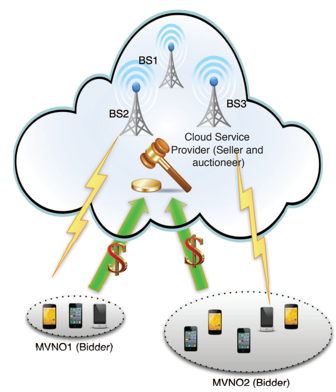

Fig. 1. Auction-based RaaS

In this paper, we introduce a novel auction-based model to enable fair pricing and fair resource allocation for RaaS. In our model as illustrated in Fig. 1, cloud service provider (i.e., seller) sells its radio resources to MVNOs. MVNOs (i.e., bidders or buyers) bid the resources according to their real-time needs and make payment to the cloud service provider. Moreover, the cloud service provider is also the auctioneer and it determines the winners among MVNOs and clears prices they should pay.

Auction mechanism design is crucial for supporting RaaS, because it directly determines the trading rules between the seller (cloud service provider) and bidders (MVNOs); furthermore it implicitly defines the behaviors of bidders. Specifically, *truthfulness* (a.k.a incentive capability or strategy-proofness) [10] and *individual rationality* [8] are desirable in RaaS auction mechanisms. An auction mechanism is truthful if a bidder will not increase its payoff by making any other bid instead of the true value. An auction lacking truthfulness could be vulnerable to market manipulation and produce very poor outcomes [5]. In addition, an auction mechanism is individually rational if the payoff of every bidder is non-negative. In this paper, based on the proposed model, we study the auction mechanism design with the objective of maximizing social welfare. First, we present an Integer Linear Programming (ILP) based auction mechanism for obtaining optimal social welfare. To reduce time complexity, we present a polynomial-time greedy mechanism for the RaaS auction. Both methods have been formally shown to be truthful and individually rational. To the best of our knowledge, we are the first to develop an auction-based model and auction mechanisms with provably-good properties for RaaS.

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II. System Model and Auction Formulation

First of all, we summarize major notations in Table I

TABLE I
NOTATIONS

Notation	Explanation
i and N	The index of BSs and the total number of BSs
j and M	The index of MVNOs and the total number of MVNOs
r_i and $\mathbf{R}$	Available dynamic resources of BS i and the corresponding vector
v_j and w_j	True valuation and declared valuation of MVNO j
$\mathbf{Y_j}$ and $\mathbf{Z_j}$	True demanded dynamic resource vector and declared dynamic resource vector of MVNO j
$\mathbf{b_j}$ and $\mathbf{B}$	Bid of MVNO j and the corresponding bid vector
x_j and $\mathbf{x}$	Winner selection variable and the corresponding vector
p_j and $\mathbf{p}$	Payment of MVNO j and the corresponding vector

We consider an RaaS cloud with N BSs and M MVNOs. We adopt the resource-based provisioning model [6] for resource sharing among MVNOs: for a BS i, an MVNO j demands a slice (in terms of percentage) of its resources so that j can provide wireless service to its mobile users that are associated with BS i. In our model, resources of BSs are allocated to MVNOs in a hybrid way (both statically and dynamically). In a statical manner, an MVNO j reserves certain percent of the total resources at each BS i (denoted by $\overline{r_{ij}}$) for a long period of time (e.g., a month or a quarter) and makes the corresponding payment in advance according to a long-term forecasting for user traffic demands based on historical data. These resources are called reserved resources and are guaranteed to be available for MVNO j. However, since both user traffic demands and link data rates are time-varying, reserved resources may not be sufficient for an MVNO for a certain short period of time. So we need to provide a way for MVNOs to request more resources from BSs according to its real-time needs. The remaining resources of BS i can be given by $r_i = 1 - \sum_{j=1}^{M} \overline{r_{ij}}$, which are referred to as *available dynamic resources* of BS i. $\mathbf{R} = (r_1, ..., r_i, ..., r_N)$ is a vector for available dynamic resources at each BS. Dynamic resource allocation is conducted periodically (e.g., once every 30min). Then the real-time demand of MVNO j at BS i can be given by $y_{ij} = \min(d_{ij} - \overline{r_{ij}}, 0)$, where d_{ij} is the fraction of resources needed by MVNO j at BS i, which can be estimated according to current link data rates and user traffic demands. $\mathbf{Y_j} = (y_{1j}, ..., y_{ij}, ...y_{Nj})$ denotes the *demanded dynamic resource vector* of MVNO j.

RaaS can be formulated as an auction mechanism design problem. *In the RaaS auction, the seller (i.e., the cloud service provider) sells available dynamic resources to bidders or buyers (i.e., MVNOs) who bid for them.* Each MVNO j is asked to declare a bid $\mathbf{b_j} = (w_j, \mathbf{Z_j})$, where w_j is the valuation and $\mathbf{Z_j} = (z_{1j}, ..., z_{ij}, ..., z_{Nj})$ is the declared dynamic resource vector. Note that the true valuation v_j and the true demanded dynamic resource vector $\mathbf{Y_j}$ are private information only known to MVNO j. So w_j and $\mathbf{Z_j}$ could be different from v_j and $\mathbf{Y_j}$ respectively. Each MVNO j is a "single-minded bidder [10]" in the sense that valuation is v_j if it gets dynamic resource no less than $\mathbf{Y_j}$ and 0 otherwise. $\mathbf{B} = (\mathbf{b_1}, ..., \mathbf{b_j}, ..., \mathbf{b_M})$ is the bid vector. We use $\mathbf{B_{-j}}$ to denote the bids of all bidders except j, so $\mathbf{B} = (\mathbf{b_j}, \mathbf{B_{-j}})$.

RaaS auction takes $\mathbf{B}$ and $\mathbf{R}$ as input, and the output includes a winner vector $\mathbf{x}(\mathbf{B}, \mathbf{R}) = (x_1, ..., x_j, ..., x_M)$ and a payment vector $\mathbf{p}(\mathbf{B}, \mathbf{R}) = (p_1, ..., p_j, ..., p_M)$. $x_j = 1$ if bidder j wins and is allocated the declared dynamic resources $\mathbf{Z_j}$; $x_j = 0$, otherwise. p_j is the payment bidder j will make to the seller. The dynamic resource allocation must satisfy the following constraints: $\sum_{j=1}^{M} z_{ij} x_j \leq r_i, \forall i \in \{1, \cdots, N\}$. Based on the output of the auction, the *payoff* [10] of bidder j is defined as

$$u_j = \begin{cases} v_j - p_j, & x_j = 1; \\ 0, & x_j = 0. \end{cases} \tag{1}$$

The *social welfare* [10] is defined as the total valuation of all winning bidders, i.e., $\sum_{j=1}^{M} v_j x_j$.

When designing an auction mechanism, it is desirable to have the following three properties [10]:

- **Individual Rationality**: an auction mechanism is *individually rational* if for any bidder j, the payoff is non-negative when bidder j bids its true value $(v_j, \mathbf{Y_j})$.
- **Truthfulness**: an auction mechanism is *truthful* if and only if for every bidder j and $\mathbf{B_{-j}}$, bidder j will not increase its payoff by making any other bid $(w_j, \mathbf{Z_j})$ instead of its true value $(v_j, \mathbf{Y_j})$; i.e., bidder j's payoff for bidding $(v_j, \mathbf{Y_j})$ is at least its payoff for bidding any other bid $(w_j, \mathbf{Z_j})$.
- **Computational Efficiency**: an auction mechanism is *computationally efficient* if the outcome can be computed in polynomial time.

Among these three properties, truthfulness is the most challenging one to achieve. In order to design a truthful auction mechanism, we introduce the following definitions.

Definition 1 (w-Monotonicity): if bidder j wins by bidding $(w_j^*, (z_{1j}^*, ..., z_{ij}^*, ..., z_{Nj}^*))$, then it also wins by bidding $(w_j', (z_{1j}^*, ..., z_{ij}^*, ..., z_{Nj}^*))$ with any $w_j' \geq w_j^*$.

Definition 2 (z-Monotonicity): if bidder j wins by bidding $(w_j^*, (z_{1j}^*, ..., z_{ij}^*, ..., z_{Nj}^*))$, then it also wins by bidding $(w_j^*, (z_{1j}', ..., z_{ij}', ..., z_{Nj}'))$ with all $z_{ij}' \leq z_{ij}^*$.

Definition 3 (Critical Payment [10]): the payment p_j for winning bidder j is set to the critical value c_j such that bidder j wins if $w_j > c_j$, and loses if $w_j < c_j$.

Lemma 1: In an RaaS auction mechanism, if w-Monotonicity, z-Monotonicity and Critical Payment are satisfied, a bidder will not increase its payoff by bidding $(v_j, \mathbf{Z_j}) = (v_j, (z_{1j}, ..., z_{ij}, ..., z_{Nj}))$ instead of $(v_j, \mathbf{Y_j}) = (v_j, (y_{1j}, ..., y_{ij}, ..., y_{Nj}))$, where $\mathbf{Y_j} \neq \mathbf{Z_j}$.

Proof: We examine two possible cases:

1) $z_{ij} < y_{ij}$ for one or more i. In this case, by bidding $(v_j, \mathbf{Z_j})$, the payoff is non-positive since the valuation is 0

when single-minded bidder j's resource demand $\mathbf{Y_j}$ cannot be met. However, the payoff of bid $(v_j, \mathbf{Y_j})$ is non-negative because if $(v_j, \mathbf{Y_j})$ is a losing bid, the payoff is 0; if $(v_j, \mathbf{Y_j})$ is a winning bid, the payoff will be non-negative.

2) $z_{ij} \geq y_{ij}$ for every i. Denote the Critical Payment for bidding $(v_j, \mathbf{Y_j})$ by p, and denote the Critical Payment for bidding $(v_j, \mathbf{Z_j})$ by p^*. Based on z-Monotonicity, we know that if a bidder loses by bidding $(v_j, \mathbf{Y_j})$, it will also lose by bidding $(v_j, \mathbf{Z_j})$. Or equivalently, for any $v_j < p$, we have $v_j < p^*$. So $p^* \geq p$. We have two sub-cases: *a)* $(v_j, \mathbf{Z_j})$ is a losing bid. In this sub-case, the payoff of bid $(v_j, \mathbf{Y_j})$ is non-negative because if $(v_j, \mathbf{Y_j})$ is a losing bid, the payoff is 0; if $(v_j, \mathbf{Y_j})$ is a winning bid, the payoff will be non-negative. *b)* $(v_j, \mathbf{Z_j})$ is a winning bid. In this sub-case, a bidder with $(v_j, \mathbf{Y_j})$ will also win and the payment will not increase. ■

Theorem 1: An RaaS auction mechanism is truthful, if it satisfies w-Monotonicity, z-Monotonicity and Critical Payment.

Proof: According to the above definition of truthfulness, we will show that a bidder will not increase its payoff by bidding any other bid $(w_j, \mathbf{Z_j}) = (w_j, (z_{1j}, ..., z_{ij}, ..., z_{Nj}))$ instead of $(v_j, \mathbf{Y_j}) = (v_j, (y_{1j}, ..., y_{ij}, ..., y_{Nj}))$. We will first show that a bidder will not increase its payoff by bidding $(w_j, \mathbf{Z_j})$ instead of $(v_j, \mathbf{Z_j})$, where $v_j \neq w_j$. Denote the Critical Payment for bidding $(v_j, \mathbf{Z_j})$ by p. We have two cases:

1) $(v_j, \mathbf{Z_j})$ is a losing bid. In this case, $v_j < p$. If a bidder with $(w_j, \mathbf{Z_j})$ loses, it would not be more beneficial than bidding $(v_j, \mathbf{Z_j})$. If a bidder with $(w_j, \mathbf{Z_j})$ wins, it makes the same payment p because the Critical Payment is independent of w_j; since $p > v_j$, the payoff of bidding $(w_j, \mathbf{Z_j})$ is negative.

2) $(v_j, \mathbf{Z_j})$ is a winning bid. If $w_j > p$, a bidder with $(w_j, \mathbf{Z_j})$ wins with the same payment p. If $w_j < p$, a bidder with $(w_j, \mathbf{Z_j})$ loses with 0 payoff.

The above two cases show that a bidder will not increase its payoff by bidding $(w_j, \mathbf{Z_j})$ instead of $(v_j, \mathbf{Z_j})$. Furthermore, in Lemma 1, we have proved that a bidder will not increase its payoff by bidding $(v_j, \mathbf{Z_j})$ instead of $(v_j, \mathbf{Y_j})$. Therefore, a bidder will not increase its payoff by bidding any other $(w_j, \mathbf{Z_j})$ instead of $(v_j, \mathbf{Y_j})$. This completes the proof. ■

III. Auction Mechanism with Optimal Social Welfare

In this section, we present a VCG-based (Vickery-Clarke-Groves[10]) auction mechanism that can achieve optimal social welfare.

A. Auction Design for Optimal Social Welfare

The RaaS auction design problem consists of two subproblems: winner selection and price determination. The winner selection problem can be formulated as the following Integer Linear Programming (ILP) problem:

ILP-Winner

$$\max \sum_{j=1}^{M} w_j x_j \tag{2}$$

Subject to:

$$\sum_{j=1}^{M} z_{ij} x_j \leq r_i, \quad \forall i \in \{1, \cdots, N\}; \tag{3}$$

$$x_j \in \{0, 1\}; \tag{4}$$

The objective is to maximize the social welfare. Constraints (3) ensure that for each BS, the sum of demanded dynamic resources does not exceed its available dynamic resources. Denote the optimal value of the ILP by $\Psi(\mathbf{B})$. Next, we present an auction mechanism that can achieve optimal social welfare, which is referred to as *Optimal-RaaS*.

(1) Winner Selection: select winners $\mathbf{x}^*$ by solving ILP-Winner;
(2) Price Determination: $p_j := \Psi(\mathbf{B_{-j}}) - (\Psi(\mathbf{B}) - w_j)$ if $x_j^* = 1$ and $p_j := 0$ otherwise. $\Psi(\mathbf{B_{-j}})$ is the optimal value of ILP-Winner with bid $\mathbf{b_j}$ removed.

B. Proof of Properties

Although Optimal-RaaS is VCG-based, the proofs of properties are non-trivial because the bids in RaaS model are multidimensional[8]. In order to prove the truthfulness of Optimal-RaaS, we show that the Winner Selection satisfies w-Monotonicity and z-Monotonicity. Furthermore, the Critical Payment condition is satisfied by the Price Determination.

Lemma 2: w-Monotonicity is satisfied in the Winner Selection of Optimal-RaaS.

Proof: Suppose that bidder j wins by bidding $\mathbf{b_j^*} = (w_j^*, (z_{1j}^*, ..., z_{ij}^*, ..., z_{Nj}^*))$. Let $\mathbf{x}$ be the winner vector. We will prove that it also wins by bidding $\mathbf{b_j'} = (w_j', (z_{1j}^*, ..., z_{ij}^*, ..., z_{Nj}^*))$ with any $w_j' > w_j^*$ by contradiction. Suppose it will lose by bidding $\mathbf{b_j'}$. Then $\Psi((\mathbf{b_j'}, \mathbf{B_{-j}})) = \Psi(\mathbf{B_{-j}})$. Since bidder j wins by bidding $\mathbf{b_j^*}$, $\Psi(\mathbf{B_{-j}}) < \Psi((\mathbf{b_j^*}, \mathbf{B_{-j}}))$. Therefore $\Psi((\mathbf{b_j'}, \mathbf{B_{-j}})) < \Psi((\mathbf{b_j^*}, \mathbf{B_{-j}}))$. Having the same winner vector $\mathbf{x}$, the social welfare with $(\mathbf{b_j'}, \mathbf{B_{-j}})$ would be greater than the social welfare with $(\mathbf{b_j^*}, \mathbf{B_{-j}})$, because $w_j' > w_j^*$; this contradicts the statement that $\Psi((\mathbf{b_j'}, \mathbf{B_{-j}})) < \Psi((\mathbf{b_j^*}, \mathbf{B_{-j}}))$. Hence the supposition is false, and bidder j will also win by bidding $\mathbf{b_j'}$. This completes the proof. ■

Lemma 3: z-Monotonicity is satisfied in the Winner Selection of Optimal-RaaS.

Proof: Suppose that bidder j wins by bidding $\mathbf{b_j^*} = (w_j^*, (z_{1j}^*, ..., z_{ij}^*, ..., z_{Nj}^*))$. Let $\mathbf{x}$ be the winner vector. We will prove that it also wins by bidding $\mathbf{b_j'} = (w_j^*, (z_{1j}^*, ..., z_{ij}', ..., z_{Nj}^*))$ with any $z_{ij}' < z_j^*$ by contradiction. Suppose it will lose by bidding $\mathbf{b_j'}$. Then $\Psi((\mathbf{b_j'}, \mathbf{B_{-j}})) = \Psi(\mathbf{B_{-j}})$. Since bidder j wins by bidding $\mathbf{b_j^*}$, $\Psi(\mathbf{B_{-j}}) < \Psi((\mathbf{b_j^*}, \mathbf{B_{-j}}))$. Therefore $\Psi((\mathbf{b_j'}, \mathbf{B_{-j}})) < \Psi((\mathbf{b_j^*}, \mathbf{B_{-j}}))$. Having the same winner vector $\mathbf{x}$, the social welfare with $(\mathbf{b_j'}, \mathbf{B_{-j}})$ is equal to $(b_j^*, \mathbf{B_{-j}})$; this contradicts the statement that $\Psi((\mathbf{b_j'}, \mathbf{B_{-j}})) < \Psi((\mathbf{b_j^*}, \mathbf{B_{-j}}))$. Hence the supposition is false, and bidder j will also win by bidding $\mathbf{b_j'}$. This completes the proof. ■

Lemma 4: $p_j = \Psi(\mathbf{B_{-j}}) - (\Psi(\mathbf{B}) - w_j)$ is a critical value for each winner bidder j in Optimal-RaaS.

Proof: In Optimal-RaaS, the payment of each winning bidder is calculated based on the opportunity cost [10], which is introduced to all the other bidders by the presence of the winning bidder. Therefore, if the bidder bids less than this price, it will not be selected as a winner, which leads to higher social welfare [14]. ■

Theorem 2: Optimal-RaaS is truthful.

Proof: According to Lemmas 2, 3 and 4 along with Theorem 1, Optimal-RaaS is truthful. ■

We then prove that Optimal-RaaS satisfies individual rationality.

Theorem 3: Optimal-RaaS is individually rational.

Proof: For any bidder j, if it bids its true value $(v_j, \mathbf{Y_j})$, its payoff is $u_j = v_j - p_j = v_j - (\Psi(\mathbf{B_{-j}}) - (\Psi(\mathbf{B}) - v_j)) = \Psi(\mathbf{B}) - \Psi(\mathbf{B_{-j}}) \geq 0$, where the last inequality follows from the optimality of $\Psi(\mathbf{B})$. This completes the proof. ■

IV. Greedy Auction Mechanism

Although Optimal-RaaS is both individually rational and truthful, it is not computationally efficient since solving ILP-Winner may take exponential time. In this section, we present an auction mechanism, called Greedy RaaS Auction Design (GRAD), which has all the three desirable properties.

A. Greedy RaaS Auction Design (GRAD)

GRAD consists of two phases too: *Winner Selection* and *Price Determination*. In the *Winner Selection* (Algorithm 1), the basic idea is to keep adding the bidder with the largest weight to the solution. We adopt the following weight α_j as the metric for sorting bidders and selecting winners:

$$\alpha_j = \frac{w_j}{\sum_{i=1}^{N} \frac{z_{ij}}{r_i}}. \tag{5}$$

In each iteration, the bidder with the maximum weight α_j is selected as the winner. Then we update $\mathbf{R}$ by subtracting the corresponding demanded dynamic resource vector $\mathbf{Z_j}$ of the selected winner from it. All the bidders who demand more dynamic resources than the available resources in the updated $\mathbf{R}$ will be eliminated from the auction. This process iterates until the bidder list is empty.

In the *Price Determination* (Algorithm 2), to find the payment for a winning bidder j, we remove j from the bidder list, do the winner selection as above with the rest bidders until a winning bidder k is found such that its selection can disqualify j from winning the auction and determine the price accordingly (lines 9–10 in Algorithm 2).

B. Proof of Properties

Lemma 5: w-Monotonicity is satisfied in the Winner Selection of GRAD.

Proof: Suppose that bidder j wins by bidding $(w_j^*, \mathbf{Z_j^*}) = (w_j^*, (z_{1j}^*, ..., z_{ij}^*, ..., z_{Nj}^*))$. We prove that it will also win by bidding $(w_j', \mathbf{Z_j^*}) = (w_j', (z_{1j}^*, ..., z_{ij}^*, ..., z_{Nj}^*))$ with any $w_j' > w_j^*$. Let $\mathbf{L}^*$ and $\mathbf{L}'$ denote the sorted lists when j bids $(w_j^*, \mathbf{Z_j^*})$ and $(w_j', \mathbf{Z_j^*})$ respectively. The positions of j in $\mathbf{L}^*$ and $\mathbf{L}'$ are denoted by q^* and q' respectively. Since $\alpha_j' = \frac{w_j'}{\sum_{i=1}^{N} \frac{z_{ij}}{r_i}} > \alpha_j^* = \frac{w_j^*}{\sum_{i=1}^{N} \frac{z_{ij}}{r_i}}$, it is clear that $q' \leq q^*$. Furthermore, at lines 9–11 in Algorithm 1, since j has not been eliminated at q^*, it will not be eliminated at q' neither. Therefore, j will still win with bid $(w_j', \mathbf{Z_j^*})$. ■

Algorithm 1: Winner Selection of GRAD

Input : Bid vector $\mathbf{B}$ and Available dynamic resource vector $\mathbf{R}$
Output: Winner vector $\mathbf{x}$

1 $x_j := 0, \forall j \in \{1, \cdots, M\}$;
2 $\alpha_j := \frac{w_j}{\sum_i \frac{z_{ij}}{r_i}}, \forall j \in \{1, \cdots, M\}$;
3 Sort the bidders in the non-increasing order of α_j and store the sorted list of their indices into $\mathbf{L}$;
4 **while** $\mathbf{L} \neq \emptyset$ **do**
5   Let j be the next bidder in $\mathbf{L}$;
6   $x_j := 1$;
7   $\mathbf{L} := \mathbf{L} \setminus \{j\}$;
8   $\mathbf{R} := \mathbf{R} - \mathbf{Z_j}$;
9   **forall the** $m \in \mathbf{L}$ **do**
10   **if** $\exists i \in \{1, \cdots, N\}$ *s.t.* $z_{im} > r_i$ **then**
11       $\mathbf{L} := \mathbf{L} \setminus \{m\}$;
12 **return** $\mathbf{x}$;

Algorithm 2: Price Determination of GRAD

Input : Bid vector $\mathbf{B}$, Available dynamic resource vector $\mathbf{R}$, Winner vector $\mathbf{x}$, Sorted bidder list $\mathbf{L}$ and weight $\alpha_j, \forall j \in \{1, \cdots, M\}$
Output: Payment vector $\mathbf{p}$

1 **forall the** $j \in \mathbf{L}$ **do**
2   $p_j := 0$;
3   **if** $x_j = 1$ **then**
4     $\mathbf{L}' := \mathbf{L} \setminus \{j\}$; $\mathbf{R}' := \mathbf{R}$;
5   **while** $\mathbf{L}' \neq \emptyset$ **do**
6   Let k be the next bidder in $\mathbf{L}'$;
7       $\mathbf{L}' := \mathbf{L}' \setminus \{k\}$;
8       $\mathbf{R}' := \mathbf{R}' - \mathbf{Z_k}$;
9   **if** $\exists i \in \{1, \cdots, N\}$ *s.t.* $z_{ij} > r_i'$ **then**
10         $p_j := (\alpha_k \sum_i \frac{z_{ij}}{r_i})$; break;
11   **forall the** $m \in \mathbf{L}'$ **do**
12   **if** $\exists i \in \{1, \cdots, N\}$ *s.t.* $z_{im} > r_i'$ **then**
13           $\mathbf{L}' := \mathbf{L}' \setminus \{m\}$;
14 **return** $\mathbf{p}$;

Lemma 6: z-Monotonicity is satisfied in the Winner Selection of GRAD.

Proof: Suppose that bidder j wins by bidding $(w_j^*, \mathbf{Z_j^*}) = (w_j^*, (z_{1j}^*, ..., z_{ij}^*, ..., z_{Nj}^*))$. We prove that it will also win by bidding $(w_j^*, \mathbf{Z_j'}) = (w_j^*, (z_{1j}^*, ..., z_{ij}', ..., z_{Nj}^*))$ with any $z_{ij}' < z_{ij}^*$. Let $\mathbf{L}^*$ and $\mathbf{L}'$ denote the sorted lists when j bids $(w_j^*, \mathbf{Z_j^*})$ and $(w_j^*, \mathbf{Z_j'})$ respectively; the positions of j in $\mathbf{L}^*$ and $\mathbf{L}'$ are denoted by q^* and q' respectively. For the sake of presentation, denote $\sum_{i=1}^{N} \frac{z_{ij}}{r_i}$ by $s(\mathbf{Z_j})$. With $z_{ij}' < z_{ij}^*$, we have $s(\mathbf{Z_j'}) < s(\mathbf{Z_j^*})$. So $\alpha_j' = \frac{w_j^*}{s(\mathbf{Z_j'})} > \alpha_j^* = \frac{w_j^*}{s(\mathbf{Z_j^*})}$; thus $q' \le q^*$. Furthermore, at lines 9–11 in Algorithm 1, since j has not been eliminated at q^*, it will not be eliminated at q' neither. Therefore, j will still win with bid $(w_j^*, \mathbf{Z_j'})$. ∎

Lemma 7: The payment p_j is set to a critical value for each winning bidder j in GRAD.

Proof: Let k be the first bidder in the list, whose selection can disqualify j. Let $c_j = \alpha_k \sum_{i=1}^{N} \frac{z_{ij}}{r_i}$. If bidder j bids $w_j < c_j$, then $\alpha_j < \alpha_k$, meaning j will be placed behind k in the sorted list and thus will be eliminated from the auction. If bidder j bids $w_j > c_j$, then $\alpha_j > \alpha_k$, meaning j will be placed ahead of k. j is ahead of any bidder that can disqualify j, because k is the first of such bidders. Therefore j will be selected as a winner and c_j is the critical value for winning bidder j. Since the payment p_j is set to c_j in the algorithm, we prove the lemma. ∎

Theorem 4: GRAD is truthful.

Proof: According to Lemmas 5, 6 and 7 along with Theorem 1, GRAD is truthful. ∎

Theorem 5: GRAD is individually rational.

Proof: We consider two possible cases: 1) Bidder j is not a winner. From Algorithm 2, j pays 0. Therefore its payoff is 0. 2) Bidder j is a winner. Since GRAD satisfies the Critical Payment property as shown in Lemma 7, we have $w_j > c_j = p_j$. In a truthful mechanism, $w_j = v_j$. Hence we have $v_j - p_j > 0$. Therefore the payoff is always non-negative. This completes the proof. ∎

Theorem 6: GRAD is computationally efficient.

Proof: In Algorithm 1, calculating α (line 2) takes $O(MN)$ time. Furthermore, the while-loop (lines 4–11) takes $O(M^2N)$ time. Hence time complexity of Algorithm 1 is $O(M^2N)$. In Algorithm 2, the for-loop takes $O(M^3N)$ time, since the for-loop (lines 1–13) runs M iterations, and in each iteration, while-loop (lines 5–13) takes $O(M^2N)$ time. So the time complexity of Algorithm 2 is $O(M^3N)$. Therefore, the overall time complexity of GRAD is $O(M^3N)$. This completes the proof. ∎

V. Performance Evaluation

We evaluated the performance of GRAD and Optimal-RaaS in terms of running time and social welfare. The simulation runs were conducted on a computer with a 2.5GHz Intel i5 CPU and 4GB memory. The social welfare is given in terms of credits, whose monetary worth can be determined by the cloud service provider (seller). In the simulation, there were a total number of 40 BSs. We evaluated the performance by varying the number of MVNOs (bidders), the demanded dynamic resources and the available dynamic resources. Specifically, we came up with the following four scenarios for our simulation: the first one for running time evaluation and the rest three for social welfare evaluation. All the numbers presented in the figures are averages over 20 runs.

1) In Scenario 1 and 2, the settings were the same. The demanded dynamic resources were uniformly distributed in $[0\%, 5\%]$; The available dynamic resources followed a uniform distribution in $[50\%, 70\%]$. The number of MVNOs was increased from 10 to 90 with a step size of 20. The corresponding results are presented in Figs. 2 and 3(a)

2) In Scenario 3, the number of MVNOs was fixed to 50; the available dynamic resources and demanded dynamic resources were uniformly distributed in $[50\%, 70\%]$ and $[0\%, u_1]$ respectively, where u_1 was increased from 3% to 7% with a step size of 1%. The corresponding results are presented in Fig. 3(b).

3) In Scenario 4, the number of MVNOs was fixed to 50; the demanded dynamic resources and available dynamic resources were uniformly distributed in $[0\%, 5\%]$ and $[50\%, u_2]$ respectively, where u_2 was increased from 50% to 90% with a step size of 10%. The corresponding results are presented in Fig. 3(c).

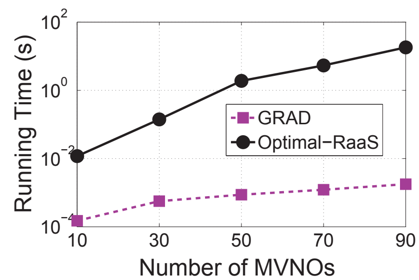


Fig. 2. Running time

We can make the following observations from these results:

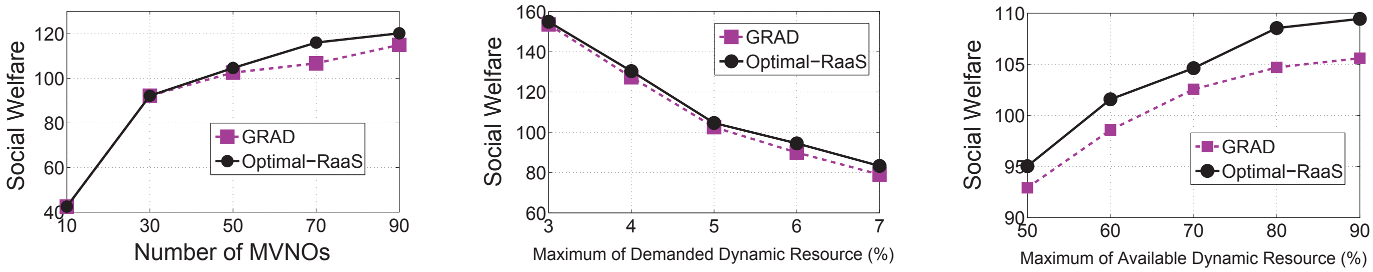
1) Fig. 2 shows the running times of the proposed mechanisms with various numbers of MVNOs. The running time of GRAD is $\frac{1}{78}$ of that of Optimal-RaaS on small cases with only 10 MVNOs. Running time savings become more and more significant when the number of MVNOs becomes larger and larger. Specifically, when it turns to 90, the running time of GRAD is about $\frac{1}{10{,}000}$ of that of Optimal-RaaS. This leads us to believe that substantial running time savings can be achieved by using GRAD.

2) Fig. 3 shows the performance of the the proposed methods with regard to social welfare. Social welfare values given by GRAD are always lower than (as expected), but close to the optimal ones. On average, by varying the number of MVNOs, the maximum demanded dynamic resource and maximum available dynamic resource, GRAD achieves 97.1%, 97.0% and 97.2% of optimal social welfare, respectively.

3) Monotonicity can be observed in Figure 3. Specifically, in Scenario 2, with more MVNOs to choose from, both methods lead to higher social welfare. In Scenario 3, more demanded dynamic resources result in fewer winning bidders (MVNOs), yielding lower social welfare. In Scenario 4, with larger available dynamic resource, more bidders are selected as winners, resulting in higher social welfare.

VI. Related Work

Cloud-based wireless networking and wireless virtualization have been studied recently. In [2], the framework CloudIQ was proposed to partition BSs into groups that are simultaneously processed on a shared homogeneous compute platform, and



(a) Scenario 2 (b) Scenario 3 (c) Scenario 4

Fig. 3. Social welfare

to schedule BSs to meet real-time processing requirements. A similar cloud-based wireless system, FluidNet, was introduced in [11]. In [4], Cudipati *et al.* introduced SoftRAN, a software defined centralized control plan for RANs that abstracts all BSs in a local geographical area as a virtual big BS. In [6], Kokku *et al.* described the design and implementation of a Network Virtualization Substrate (NVS) for effective virtualization of wireless resources in cellular networks. Their follow-up work can be found in [7, 9].

All these related works studied how to enable virtualization in specific wireless networks. We, however, present a general auction-based model and mechanisms for operating RaaS, assuming virtualization is enabled on BSs.

The auction theory has been studied for decades. Vickery (1961) [12] proposed the notion of truthful bidding in a sealed-bid auction, and introduced the second-price auctions. Clarke and Groves extended his work, yielding the famous Vickery-Clarke-Groves (VCG) mechanism [10]. It has been proved in [10] that every VCG mechanism is truthful (incentive compatible). In the meanwhile, to mitigate high time complexity of VCG, some works were focused on proposing fast greedy heuristic algorithms without sacrificing truthfulness [1, 14].

Recently, efforts have been made to apply the auction theory to support network virtualization. In [3], the interactions among Service Providers (SP) and Network Provider (NP) were modeled as a stochastic game; each stage of the game is played by SPs (on behalf of end users) and is regulated by the NP through a VCG mechanism. In [16], with a non-cooperative game model, Zhou *et al.* presented a bandwidth allocation scheme with Nash Equilibrium for a virtualized network environment. In [15], a truthful and computationally efficient spectrum auction named VERITAS was proposed to support eBay-like dynamic spectrum market with the objective of maximizing spectrum utilization. Our paper represents the first work to study auction design for RaaS, which is mathematically different from those problems considered in the related works.

VII. Conclusion

In this paper, we proposed a novel auction-based model for RaaS. Based on the proposed model, we studied the auction mechanism design with the objective of maximizing social welfare. First, we proposed Optimal-RaaS, which is an ILP-based auction mechanism that can achieve optimal social welfare. To reduce time complexity, we proposed GRAD, which is a polynomial-time greedy mechanism for the RaaS auction. Both methods have been formally shown to be truthful and individually rational. Extensive simulation results show that GRAD can quickly produce close-to-optimal solutions.

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